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A MODIFIED DECOMPOSITION SOLUTION OF TRIANGULAR FIN WITH TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY AND HEAT GENERATION

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Abstract
In this paper, we propose, the modified decomposition method (MADM) is a recent mathematical technique for a closed form solution of nonlinear heat transfer equation of a longitudinal triangular fin where the thermal conductivity and internal heat generation are temperature dependent. The energy balance equation of triangular fin with the temperature dependent thermal conductivity and internal heat generation is solved by MADM. The results obtained from the MADM are compared with the differential transform method (DTM). The effects of the various thermo physical parameters, such as convective-conductive parameter, heat generation number, and convective sink temperature on the dimensionless temperature distribution, are analyzed comprehensively.
Keywords: Modified decomposition method; triangular fin; singular value problem; modified differential operator;

Nomenclature

C Constant which represents the temperature
k Temperature dependent thermal conductivity, $W/(mK)$
k_a Thermal conductivity corresponding to ambient condition, $W/(mK)$
T Temperature, K
P Fin perimeter, m
T_b Fin's base temperature, K
T_a Sink temperature corresponding to k_a, K
L Length of the fin, m
x Axial co-ordinate of the entire fin, m
A_c Cross-sectional area of the entire fin, m^2
X Dimensionless axial co-ordinate
A Thermal conductivity parameters
ε_G Heat generation parameters
G Heat generation number
t_b base thickness of the fin

Greek symbols

α Slope of the thermal conductivity-temperature curve, K^{-1}
γ Slope of the heat generation-temperature curve, K^{-1}

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θ Dimensionless temperature of the fin,
θ_a Dimensionless sink temperature of the fin corresponding to k_a,
ρ Density of the fin materials

1. Introduction

Extended surfaces are the effective and economical means of enhancing the heat transfer rate between the heated surfaces and surrounding environment. An extensive review on this topic is presented by Kraus et.al [1].There is no doubt that the rectangular fin is simple in shape but its weight is relatively more as compare to the parabolic,exponential and triangular profile. Therefore with the passage of time the material utilization and light weight is becoming paramount factor particularly for airborne and space application. Therefore various fin profile involving, triangular, parabolic and exponential profiles came into picture.
However the flow of electric current or atomic reaction are the sometimes are the causes of internal heat generation. Aziz and Bouaziz [2] used optimal linearization method to study the performance and design criteria of rectangular fin with variable thermal conductivity and heat generation. Das and Singla [3] predicted geometry and heat generation number of rectangular fin with variable thermal conductivity and temperature dependent internal heat generation using Adomian decomposition method.
From the forgoing discussion it is clear that numerous contributions have been made for solving the fin with variable parameters with the classical ADM and other semi analytical methods. Modified decomposition method (MADM) is a small modification on ADM have been developed and introduced by Georged Adomian in 1984 gives a converge solution rapidly [4].

2. Mathematical Model and Formulations

The present study considers a triangular fin profile having length L, width W, and base thickness t_b dissipating heat to the environment through both convection with some internal heat source. The cross-sectional area of fin changing linearly from fin tip to the base according to $A(X) = t_b WX$ with perimeter P, where X is the non dimensional length measured from the fin tip to the base. The fin width is very large as compared to its thickness so that $P \approx W$. The effect of environmental temperature can be varied independently.
The one dimensional steady state energy equation for the triangular fin can be expressed as

$$\frac{d}{dx}\left[k(T)A(x)\frac{dT}{dx}\right] - hPA(x)(T - T_a) + q(T)A(x) = 0$$

(1)

Since the material undergoing the thermal treatment experience a large change in its temperature during the process, the thermal conductivity, and internal heat source of the fin material are linear function of temperature.

$$k(T) = k_a\left[1 + \alpha(T - T_a)\right]$$

2(a)

$$q(T) = q_0\left[1 + \gamma(T - T_a)\right]$$

(2b)

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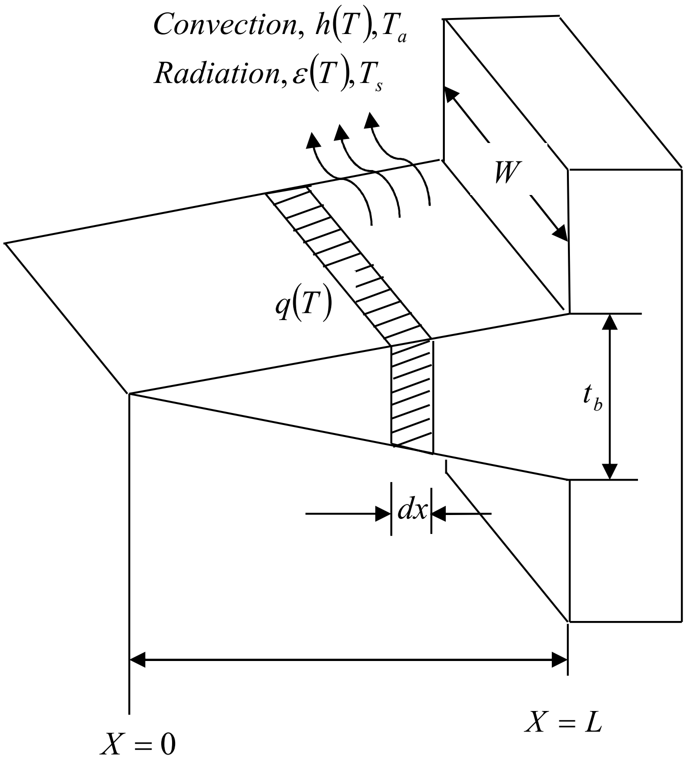

Figure 1. The geometry of straight triangular fin.

With the insulated boundary conditions

$$\frac{dT}{dx} = 0 \quad at \quad x = 0 \tag{3a}$$

$$T = T_b \quad at \quad x = L \tag{3b}$$

The non-dimensional form of energy equation (1) for the convex fin with the relevant boundary conditions can be expressed as below

$$\frac{d}{dX}\left[\{1 + A(\theta - \theta_a)\}X\frac{d\theta}{dX}\right] - N_c(\theta - \theta_a) + GX[1 + \varepsilon_G(\theta - \theta_a)] = 0 \tag{4}$$

With the boundary condition

$$\frac{d\theta}{dX} = 0 \quad at \quad X = 0 \tag{5a}$$

$$\theta = 1 \quad at \quad X = 1 \tag{5b}$$

Where the dimensionless number are shown in Table 1

Table 1 Dimensionless term used in energy equation of triangular fin

Dimensionless group	Definition

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Dimensionless temperature $\theta = \dfrac{T}{T_b}$

Dimensionless convective environmental temperature $\theta_a = \dfrac{T}{T_a}$

Dimensionless axial coordinate temperature $X = \dfrac{x}{L}$

Convective conductive parameter $N_c = \dfrac{h_b L^2}{k_a t_b}$

Dimensionless heat generation number $G = \dfrac{q_0 L^2}{k_a T_b}$

Thermal conductivity parameter $A = \alpha T_b$

Internal heat generation parameter $\varepsilon_G = \gamma T_b$

3. Methodology of MADM

$$U^{(n+1)} + \frac{m}{X} U^{(n)} + Nu = g(X) \tag{6}$$

$U(0) = a_0$

$U^{'}(0) = a_1$

$U^{"}(0) = a_2$

M

$U^{n-1}(0) = a_{n-1}$

$U(b) = c$

Where N is the non linear differential operator of order less than n, $g(X)$ is the given function and $a_0, a_1, a_2 \Lambda\ a_{n-1}, b \,\&\, c$ are given constants.

We propose the new differential operator as below

$$L(\bullet) = X^{-1} \frac{d^{n-1}}{dX^{n-1}} X^{n-m} \frac{d}{dX} X^{m-n+1} \frac{d}{dX}(\bullet) \tag{7}$$

So the equation (6) can be written as

$$LU + NU = g(X) \tag{8}$$

The inverse operator L^{-1} is therefore considered a $(n+1)$ fold integral operator as below

$$L^{-1}(\bullet) = \int_0^X X^{n-m-1} \int_0^X X^{m-n} \int_0^X \Lambda \int_0^X X(\bullet) dX dX$$

Applying inverse operator on both sides of the equation (8)

$$L^{-1}LU + L^{-1}NU = L^{-1}g(X) \tag{9}$$

Which can written as

$$U = a_1 + a_2 X + L^{-1} g(X) - L^{-1} NU \tag{10}$$

The energy equation (4) of the convex profile can be written as

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$$\frac{d^2\theta}{dX^2}+\frac{1}{X}\left(\frac{d\theta}{dX}\right)=-G+G\varepsilon_G\theta_a-G\varepsilon_G\theta+\frac{N_c(\theta-\theta_a)}{X}+A\theta_a\frac{d^2\theta}{dX^2}-A\left(\frac{d\theta}{dX}\right)^2$$
$$-A\theta\frac{d^2\theta}{dX^2}+\frac{A\theta_a}{X}\left(\frac{d\theta}{dX}\right)-\frac{A}{X}\left(\theta\frac{d\theta}{dX}\right)$$
(11)

We propose new modified differential operator for the above equation is

$$L_{XX}(\bullet)=X^{-1}\frac{d}{dX}X\frac{d}{dX}(\bullet) \tag{12}$$

And new modified differential inverse operator for the above equation is given by

$$L_{XX}^{-1}(\bullet)=\int_0^X X^{-1}\int_0^X X(\bullet)dXdX \tag{13}$$

The energy equation (9) of the convex fin in operator form

$$L_{XX}\theta=-G+G\varepsilon_G\theta_a-G\varepsilon_G\theta+\frac{N_c(\theta-\theta_a)}{X}+A\theta_a\frac{d^2\theta}{dX^2}-A\left(\frac{d\theta}{dX}\right)^2-A\theta\frac{d^2\theta}{dX^2}$$
$$+\frac{A\theta_a}{X}\left(\frac{d\theta}{dX}\right)-\frac{A}{X}\left(\theta\frac{d\theta}{dX}\right)$$
(14)

In the above equation $\left(\frac{d\theta}{dX}\right)^2$, $\theta\frac{d\theta}{dX}$,and θ , $\frac{d^2\theta}{dX^2}$, $\frac{d\theta}{dX}$,are the non linear and linear terms respectively. Next by expressing nonlinear and linear terms by Adomian polynomials and by applying the operators the energy equations is modified in the following form.

$$L_{XX}\theta=-G+G\varepsilon_G\theta_a-G\varepsilon_G\theta+\frac{N_c(ND)}{X}+A\theta_a(NE)-A(NF)-A(NG)+\frac{A\theta_a}{X}(NH)-\frac{A}{X}(NI)$$
(15)

Where ND ,NE , NF, NG, NI and are the non linear and linear expressions to be expanded in terms of Adomian polynomials. These abdominals polynomials are expressed as below.

Applying inverse operators $\left(L_{XX}^{-1}\right)$, on both sides of the equation (13), the following expressions are obtained.

$$\theta=\theta_0-GL_{XX}^{-1}(1)+G\varepsilon_G\theta_aL_{XX}^{-1}(1)-G\varepsilon_GL_{XX}^{-1}\left(\sum_0^\alpha\theta_m\right)+N_CL_{XX}^{-1}\left(\frac{\sum_0^\alpha D_m}{X}\right)+A\theta_aL_{XX}^{-1}\left(\sum_0^\alpha E_m\right)-$$
$$AL_{XX}^{-1}\left(\sum_0^\alpha F_m\right)-AL_{XX}^{-1}\left(\sum_0^\alpha G_m\right)+A\theta_aL_{XX}^{-1}\left(\frac{\sum_0^\alpha H_m}{X}\right)-AL_{XX}^{-1}\left(\frac{\sum_0^\alpha I_m}{X}\right)$$
(16)

Where $\theta_0=\theta(0)+X\frac{d\theta(0)}{dX}$ .Assuming some finite tip temperature of the triangular fin that implies whole θ_o component should be replaced with arbitrary constant C which can be written as below

$$\theta_o=C$$

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Considering a finite series of order m, the higher order terms in the above equation (14) are obtained recursively as

$$\theta_{m+1} = -GL_{XX}^{-1}(1) + G\varepsilon_G\theta_a L_{XX}^{-1}(1) - G\varepsilon_G L_{XX}^{-1}\left(\sum_0^{\alpha}\theta_m\right) + N_C L_{XX}^{-1}\left(\frac{\sum_0^{\alpha} D_m}{X}\right) + A\theta_a L_{XX}^{-1}\left(\sum_0^{\alpha} E_m\right)$$
$$- AL_{XX}^{-1}\left(\sum_0^{\alpha} F_m\right) - AL_{XX}^{-1}\left(\sum_0^{\alpha} G_m\right) + A\theta_a L_{XX}^{-1}\left(\frac{\sum_0^{\alpha} H_m}{X}\right) - AL_{XX}^{-1}\left(\frac{\sum_0^{\alpha} I_m}{X}\right) \quad with\ \ m \ge 0 \tag{17}$$

In the present analysis the estimation of the first three significant terms, i.e., m=0 to 2, of the temperature field is now expressed as in the following form

$$\theta_1 = -GL_{XX}^{-1}(1) + G\varepsilon_G\theta_a L_{XX}^{-1}(1) - G\varepsilon_G L_{XX}^{-1}\theta_0 + N_C L_{XX}^{-1}\left(\frac{D_0}{X}\right)$$
$$+ A\theta_a L_{XX}^{-1}E_0 - AL_{XX}^{-1}F_0 - AL_{XX}^{-1}G_0 + A\theta_a L_{XX}^{-1}\left(\frac{H_0}{X}\right) - AL_{XX}^{-1}\left(\frac{I_0}{X}\right) \tag{18}$$

$$\theta_2 = -G\varepsilon_G L_{XX}^{-1}\theta_1 + N_C L_{XX}^{-1}\left(\frac{D_1}{X}\right) + A\theta_a L_{XX}^{-1}E_1 - AL_{XX}^{-1}F_1 - AL_{XX}^{-1}G_1 + A\theta_a L_{XX}^{-1}\left(\frac{H_1}{X}\right) - AL_{XX}^{-1}\left(\frac{I_1}{X}\right) \tag{19}$$

$$\theta_3 = -G\varepsilon_G L_{XX}^{-1}\theta_2 + N_r L_{XX}^{-1}\left(\frac{A_2}{X}\right) + N_C L_{XX}^{-1}\left(\frac{D_2}{X}\right) + A\theta_a L_{XX}^{-1}E_2 - AL_{XX}^{-1}F_2 - AL_{XX}^{-1}G_2$$
$$+ A\theta_a L_{XX}^{-1}\left(\frac{H_2}{X}\right) - AL_{XX}^{-1}\left(\frac{I_2}{X}\right) \tag{20}$$

Therefore finally temperature field can be obtained in terms of finite series.

$$\theta = \sum_0^m \theta_m = \theta_0 + \theta_1 + \theta_2 + \theta_3 + \Lambda \tag{21}$$

5. Results and discussion

Figure 2 demonstrates the validation MADM with the DTM for other parameters at fixed level. It has been observed that the present MADM results are good agreement with DTM as available in the literature.

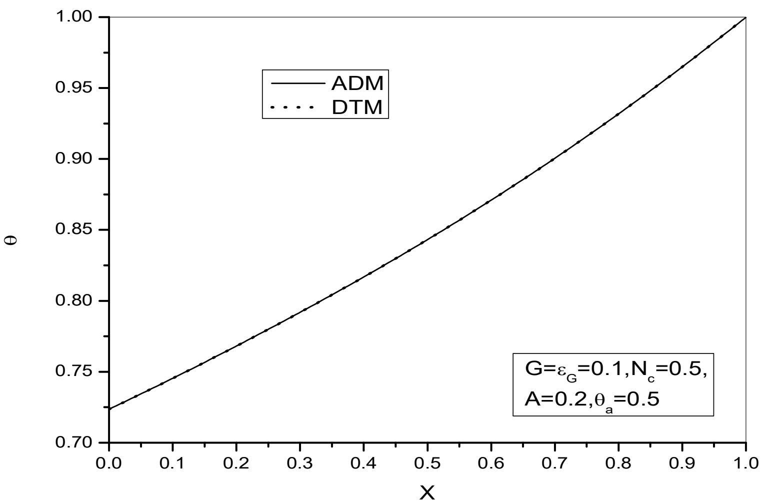

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Figure 2. Comparison of Modified decomposition method (MADM) with differential transform method (DTM).

Figure 3 shows the variation of dimensionless temperature with the dimensionless length of longitudinal triangular fin. The figure displays that temperature in the fin increases with the increasing value of thermal conductivity parameter (A). The bottom line corresponds to constant thermal conductivity line (A=0) and gives a constant temperature gradient throughout the length of the triangular fin. Physically the effect of increase of thermal conductivity parameter enhances the heat conduction process and results in an increase in the dimensionless temperature of the triangular fin. As the value of thermal conductivity parameter increases the temperature gradient slope increases from the fin tip to the base and highest slope is observed in case of curves corresponds to A=1.

Figure 4 shows the variation of dimensionless temperature with the dimensionless length and also the effect of variation of thermo geometric fin parameter on the straight triangular fin.

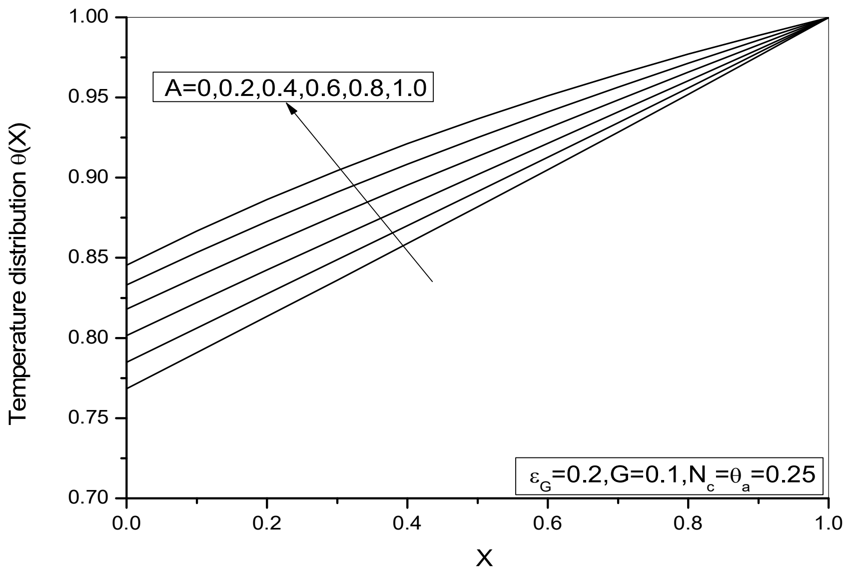

Figure 3. The dimensionless temperature distribution for varying values of A when ε_G=0.2,G=0.1,Nc=θ_a = 0.25

The value of thermogeometric fin parameter Nc indicates the relative strength of convection versus conduction. If the value of N_c=1, the convection and conduction are at equal rates and when for N_c=0, the convection mode of heat transfer is zero. The top curves corresponds to the heat transfer of triangular fin without convection (N_c=0) and bottom curves corresponds to the triangular fin with relatively higher value of convection heat transfer rates (N_c=1.25). As the thermo geometric fin parameter increases the rate of heat transfer through the fin increases and producing lower dimensionless temperature. The temperature in the fin drops faster and the temperature profiles become steeper that results higher heat transfer rate through the base of the fin.

Figure 5. Shows the variation of convection sink temperature on the dimensionless temperature distribution with respect to the dimensionless length of moving fin. The environmental temperature is important parameter for selecting the processing time of the fin. Higher environmental temperature takes small time to release heat in environment. The bottom curves corresponds to the fin whose convection sink maintained at absolute zero. As the convective sink temperature increases, the convective heat loss from the triangular fin decreases that produce higher dimensionless temperature in the fin.

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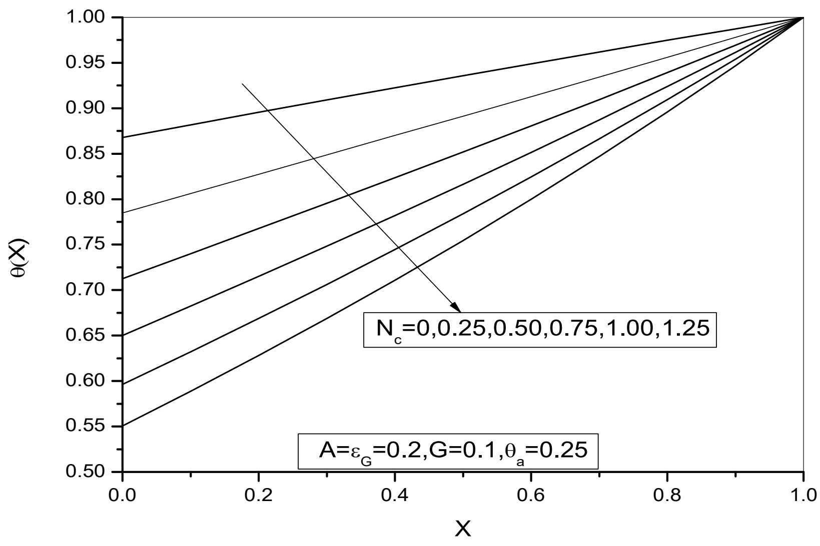

Figure 4. The dimensionless temperature distribution for varying values of thermo geometric fin parameter N_c when A=ε_G=0.2, G=0.1, θ_a = =0.25.

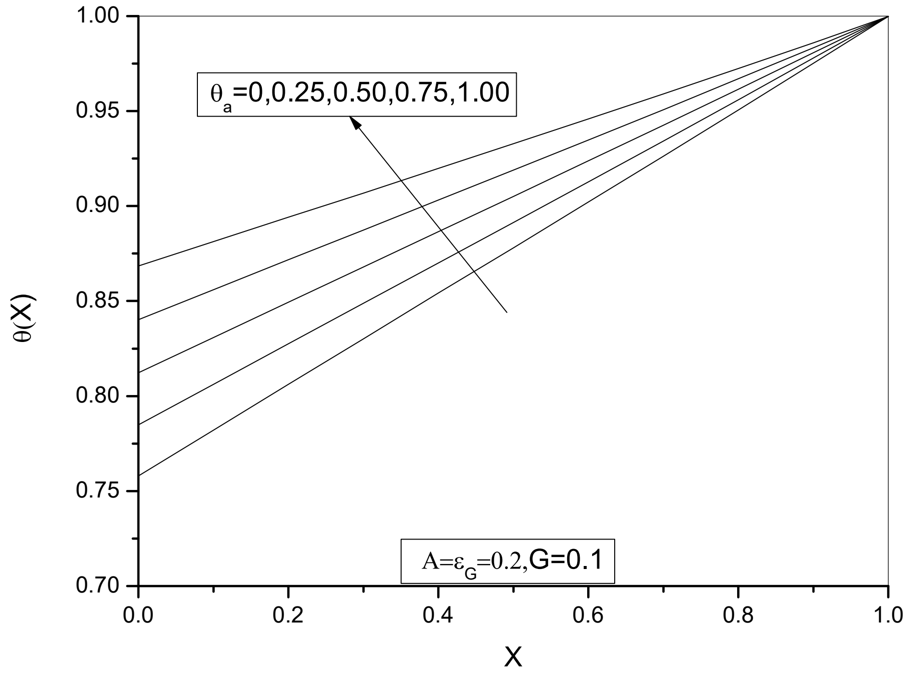

Figure 5. The dimensionless temperature distribution for varying values of θ_a when A= ε_G =0.2, G=0.1.

6.Conclusions

The closed form solution for dimensionless temperature field of a longitudinal triangular fin with variable thermal conductivity and internal heat generation is obtained using modified decomposition method (MADM).This method explicitly solves the complex heat transfer equation

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with temperature dependent thermal conductivity and internal heat generation. The accuracy of the solution is verified by comparing with the available results with DTM solution. The results are found to be fairly good for many values of thermal parameters and heat transfer processes with similar boundary conditions.

References

[1] Kem D.Q, Kraus D.A. Extended Surface Heat Transfer. McGraw–Hill, New York, 1972.

[2] A.Aziz,M.N.Bouaziz,A lest square method for a longitudinal fin with temperature dependent internal heat generation and thermal conductivity, Energy conversion and Management, **2011**, 52, 2876-2882.

[3] R.K. Singla, Ranjan Das, Application of Adomian decomposition method and inverse solution for a fin with variable thermal conductivity and heat generation, International Journal of Heat and Mass transfer 66,2013, 496-506.

[4] Adomian G. Solving frontier problems in physics: the decomposition method. Dordrecht: Kluwer Academic Publishers; 1994.