International Journal of Engineering Technology and Management Sciences
Website: ijetms.in Issue: 1 Volume No.8 January - February – 2024
DOI:10.46647/ijetms.2024.v08i01.031 ISSN: 2581-4621

A Comparative Stability Analysis of Inverted Pendulum using MIT Rule, Fractional-Order MIT Rule and Modified MIT Rule-based MRAC

Dhananjay Gupta[1*], Awadhesh Kumar[2], Vinod Kumar Giri[3]
[1] *Research Scholar – Electrical Engineering, MMMUT, Gorakhpur, Uttar Pradesh*
[2] *Assistant Professor - Electrical Engineering, MMMUT, Gorakhpur, Uttar Pradesh*
[3] *Professor - Electrical Engineering, MMMUT, Gorakhpur, Uttar Pradesh*
**Corresponding Author Orcid ID: 0000-0001-6205-8417*

ABSTRACT
In control engineering, Inverted Pendulum Systems (IPS) stability analysis is crucial. In this paper, a normal MIT (Massachusetts Institute of Technology) rule, fractional-order MIT (FOMIT) rule and modified MIT rule-based Model Reference Adaptive controller (MRAC) have been designed to stabilize IPS. The nature of the performance of IPS has also been analyzed to track a stable reference model. After implementation of all the three mentioned methods, it has been found that MRAC using the standard MIT rule is unable to control IPS and FOMIT rule-based MRAC method necessitates higher values of adaptation-gains to achieve the desired response, while modified MIT rule-based MRAC shows the desired response at reduced adaptation gain value. The performance analysis has been carried out by comparing the results obtained for all the three mentioned rules with the variations in the adaptation gain, rise time, settling time and peak overshoot on MATLAB/Simulink platform. The obtained results present encouraging outcomes.
Keywords— MRAC, Adaptation gain, Reference Model, MIT Rule, Fractional-Order MIT Rule, Modified MIT rule

1. Introduction

An inverted pendulum system is an excellent example of a modern control challenge because it is a very unstable nonlinear system [1]. It belongs to the category of less triggering systems, where the number of mechanical control inputs is smaller than the number of degrees of freedom. This adds to the difficulty of the IPS in planning, testing, and judging different classical control methods. As a result, researchers are now curious about how to regulate an Inverted Pendulum (IP). It provides the ideal setting for the testing and implementation of several logics in the field of control engineering [2]. Ship stability in contrast to winds, regulating the altitude of aircraft, missiles, airplane landing, etc., includes regulating behavior comparable to the IP control [3].

Several control techniques have been used to illustrate the IP, including linear techniques such as state space pole placement control [4], linear quadratic regulator (LQR) method [5-6] and Proportional Integral Derivative (PID) control [7], Hedge-algebras theory and nonlinear methods such as robust control [8], energy-based and passive-control [9].

The aim of this work is to stabilize and study the nature of the performance of IPS with different types of MIT rule-based MRAC. The stability analysis of an inverted pendulum using the standard MIT rule, the fractional-order MIT rule, and the modified MIT rule on MRAC to follow a stable reference model has been conducted. In conclusion, MRAC based on the standard MIT rule is ineffective at regulating IPS, while the FOMIT rule-based MRAC strategy requires greater values of adaptation gain. At lower adaptation gain values, MRAC based on a modified MIT rule is able to achieve the desired response.

DOI:10.46647/ijetms.2024.v08i01.031 ISSN: 2581-4621

2. Inverted Pendulum

The inverted pendulum has been used as a standard for evaluating control strategies since it is one of the most challenging systems to stabilize in the area of control-systems [10]. In a nonlinear dynamic system, an IPS has a stable balance point in the pending situation and an unstable one when it is upright. In our research, we have assumed the inverted pendulum shown in Fig.1.

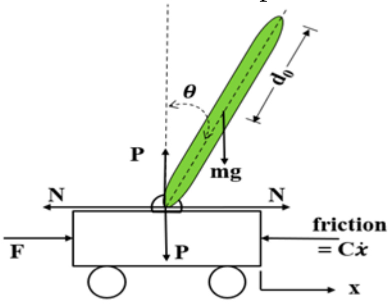


Fig.1: Free body diagram of the inverted pendulum system

The pendulum consists of weight dangling from a pivot so that it can swing back and forth. A lateral displacement of the pendulum's equilibrium points experiences a reinstating force owing to gravity that causes it to speed up and return to that position at an angle of θ = 0. When the pendulum's mass and the reinstating force are assorted, the outcome is a back-and-forth motion about the balance point. In the above Fig. 1, let d1 be the length of the pendulum, C is the frictional constant, m is the mass of the pendulum, g is acceleration, d0 is the half-length of the pendulum at the centre, and T is the tension. The pendulum's equation of motion is [11]:

$$J\frac{d^2}{dt^2} + C\frac{d\theta}{dt} - mgd_0\cos\theta = (d_1)T \tag{1}$$

Taking Laplace Transform of eq. (1) rearranging as

$$\frac{\theta(s)}{T(s)} = \frac{d_1}{Js^2 + Cs - mgd_0} \tag{2}$$

The parameters of IPS are given in Table 1:

Table 1: Plant Parameters Values [11]

Parameters	Values
Inertia (J)	0.2453 Kg.m2
Frictional Force (C)	0
Mass of Pendulum (m)	900 g
Gravitational Acceleration (g)	9.81 m/s2
Half Length of the Pendulum at the Centre (d0)	0.051 m
Length of Pendulum (d1)	0.102 m

Parameter values for a real-time process lead to the following transfer function for the entire system, given by eq. (3)

$$\frac{\theta(s)}{T(s)} = \frac{0.102}{0.2453s^2 - 0.4503} \tag{3}$$

It may also be written as eq. (4),

$$\frac{\theta(s)}{T(s)} = \frac{0.4158}{s^2 - 1.836} \tag{4}$$

3. Model Reference Adaptive Control

MRAC is an example of non-dual adaptive control, which is a larger class of control systems. [12]. We can find out how well the system works by looking at a reference model. A difference method is used to modify the parameters of the feedback controller after comparing the real output with the modelled output. In order to approximate the reference model, the MRAC models the output of the plant or system. Fig. 2 shows the fundamental block diagram of the MRAC system.

DOI:10.46647/ijetms.2024.v08i01.031 ISSN: 2581-4621

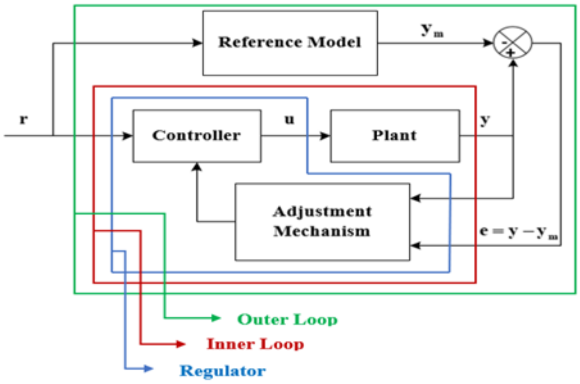


Fig.2: Block diagram of Model Reference Adaptive Controller

The requirement-based reference model selection is the initial stage in MRAC. The next step is to update the controller's configurable settings by designing the control algorithm. The intended performance parameters, including the system response's peak-overshoot (M_p), settling time (T_s), and rise time (T_r), are detailed in the reference model. Based on Dinakin and Oluseyi's (2021) study, this research work employs a critically damped (ζ=1) second-order system with a natural frequency (ω_n=3) as reference model. The reference model's transfer function is stated as [13]:

$$\frac{y_m(s)}{r(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \quad (5)$$

$$\frac{y_m(s)}{r(s)} = \frac{9}{s^2+6s+9} \quad (6)$$

4. MIT Rule

People often refer to this rule as the MIT rule since it was developed by the Massachusetts Institute of Technology (MIT). It uses practical systems to implement the MRAC framework. A loss function J, often called the cost function, was required for the system stability analysis according to the MIT rule. This function can be represented as [14, 15]:

$$J(\theta) = \frac{1}{2}e^2 \quad (7)$$

$$\frac{\partial J}{\partial e} = e \quad (8)$$

This is where e stands for output error, which is the variation between the plant's output and the reference model's output, and θ (i.e., θ1 and θ2) is the control parameter, commonly known as the regulating parameter. A parameter represented by θ (i.e., θ1 and θ2) is adjusted to decrease the loss function in this scenario. Hence, it would be suitable to change the value such that it moves counter to J's gradient. Fig. 3 shows the MRAC block diagram that follows the MIT rule. Here we show the most fundamental form of the MIT rule [13]:

$$\frac{d\theta}{dt} = -\gamma\frac{\partial J(\theta)}{\partial\theta} \quad (9)$$

$$\frac{d\theta}{dt} = -\gamma e\frac{\partial e(\theta)}{\partial\theta} \quad (10)$$

The sensitivity derivative of a plant is represented by ∂e/∂θ. The symbol θ represents the parameter and this term shows how changing it impacts the error. The γ is adaptation gain [11, 13]. A tuning parameter that controls the rate of adaptation in the controller is called adaptation gain. Finding an appropriate balance between rapid convergence and stability is the goal of the adaptation gain. Faster convergence and parameter updates are possible with a large adaption gain, but the control system is more likely to become unstable and overshoot if this gain is too high. While a modest adaption gain may enhance stability, it may also cause convergence to be slower and tracking performance to be worse. Both the properties of the controlled system and the performance requirements should be considered when choosing the adaption gain. A systematic approach, like optimisation algorithms, advanced control design methodologies, or trial and error, can be used to find the optimal adaptation gain. To determine an appropriate range of adaptation gain, this paper use the trial-and-error technique. The control law is described as (11).

International Journal of Engineering Technology and Management Sciences
Website: ijetms.in Issue: 1 Volume No.8 January - February – 2024
DOI:10.46647/ijetms.2024.v08i01.031 ISSN: 2581-4621

$$u = \theta_1 r - \theta_2 y \tag{11}$$

where θ1 and θ2 are controller parameters. The Inverted Pendulum system's transfer function is obtained as,

$$\frac{\theta(s)}{T(s)} = \frac{0.4158}{s^2 - 1.836} \tag{12}$$

The above eq. (12) may be simplified as

$$\frac{y(s)}{u(s)} = \frac{b}{s^2 - a} \tag{13}$$

By substituting the control law from the eq. (11) in eq. (13), we obtain,

$$\frac{y(s)}{r(s)} = \frac{b\theta_1}{s^2 - a + b\theta_2} \tag{14}$$

By comparing the above eq. (14) with eq. (5) we get,

$$\theta_1 = \frac{\omega_n^2}{b} \quad \text{and} \quad \theta_2 = \frac{\omega_n^2 + a}{b} \tag{15}$$

To achieve optimal model tracking, the controller parameters must converge to these values. The difference between the output of the reference model, y_m, and the plant output, y, is defined as the term "error function," It can be represented as follows by eq. (16):

$$e = y - y_m \tag{16}$$

We are taking Laplace Transform of eq. (16) and putting eq. (5) and eq. (14), we get:

$$e = \frac{b\theta_1}{s^2 - a + b\theta_2} r - \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} r \tag{17}$$

$$\frac{\partial e}{\partial \theta_1} = \frac{b}{s^2 - a + b\theta_2} r \tag{18}$$

$$\frac{\partial e}{\partial \theta_2} = \frac{-b^2\theta_1}{s^2 - a + b\theta_2} r \tag{19}$$

By putting the value of θ2 from eq. (15) in eq. (18) and (19), we get,

$$\frac{\partial e}{\partial \theta_1} = \frac{b}{s^2 + \omega_n^2} r \tag{20}$$

$$\frac{\partial e}{\partial \theta_2} = \frac{-b}{s^2 + \omega_n^2} y \tag{21}$$

By substituting the eq. (20) and (21) in eq. (10), we obtain eq. (22) and (23), respectively,

$$\frac{\partial \theta_1}{\partial t} = -\gamma e \frac{b}{s^2 + \omega_n^2} r \tag{22}$$

$$\frac{\partial \theta_2}{\partial t} = \gamma e \frac{b}{s^2 + \omega_n^2} y \tag{23}$$

Where γ is adaptation gain for controller parameters θ1 and θ2, respectively.

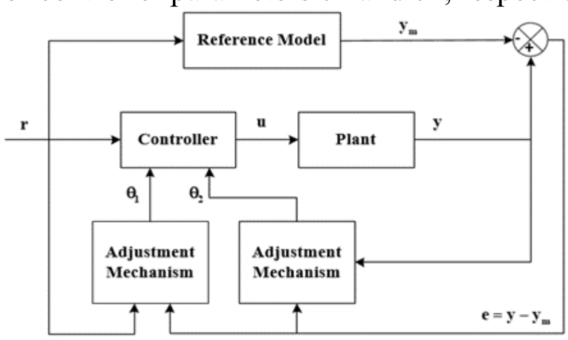


Fig. 3: Block diagram of MRAC using standard MIT rule

5. Fractional-Order MIT Rule

As a result of the MIT rule, the controller is extremely sensitive to variations in the amplitude of the reference input. An improved technique [11] of the MIT rule for adjusting parameters to generate the control law has been presented to address this issue. Here, we apply the MIT rule expression to the G-L (Grünwald-Letnikov) fractional-derivative [11] on the error signal, and the resulting equation looks like eq. (24),

$$\frac{d\theta}{dt} = -\gamma e \frac{d^\alpha e}{d\theta^\alpha} \tag{24}$$

DOI:10.46647/ijetms.2024.v08i01.031 ISSN: 2581-4621

Both adaption gain and derivative order alpha contribute to the rate of change of the parameter θ. So, the definition of the G-L fractional derivative [16, 17] is

$$D^{\alpha} f(t) = lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{k=0}^{n} (-1)^k \binom{n}{k} f(kh - h) \quad (25)$$

In which h is the step size. It has been derived under the assumption that $D^{\alpha} f(t) \approx D_h^{\alpha} f(t)$,

$$D_h^{\alpha} f(t) = h^{-\alpha} \sum_{j=0}^{k} (-1)^j \binom{\alpha}{j} f(kh - jh) \quad (26)$$

Now $\binom{\alpha}{j}$ can be approximated as

$$\frac{\alpha!}{j!(\alpha-j)!} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)} \quad (27)$$

Here Γ is a gamma function [13]. The error signal has been processed by this gamma function.

$$\frac{d\theta}{dt} = -\gamma \left(\frac{d^{\alpha} e}{d\theta^{\alpha}}\right) y_m \quad (28)$$

In this equation, γ is the adaptive gain, ym is the output of reference mode, e is the error between the plant and reference model, and α is the extra degree of freedom. This fractional-order rule was developed in MATLAB Simulink using these mathematical equations

6. Modified MIT Rule

Here, we go over how the IPS has adopted an MRAC scheme based on modified MIT rules. The modified MIT rule-based MRAC is planned to improve the response of the system. The modified MIT rule-based MRAC scheme is just the PID controller superimposed on the MIT rule-based MRAC control method. Here, we stabilize the system and follow the intended response by combining the control laws of MRAC using the MIT rule and PID control law. The PID control law's Proportional, Integral, and Derivative gains can be modified with the MIT rule's adaption parameters. Therefore, the controller law is demonstrated as Eq. (29). Fig. 4 shows the block diagram of the modified MIT rule.

$$u = \theta_1 r - \theta_2 y - \left(k_p e + k_i \int e dt + k_d \frac{de}{dt}\right) \quad (29)$$

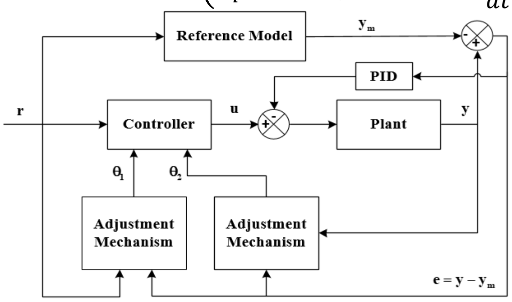
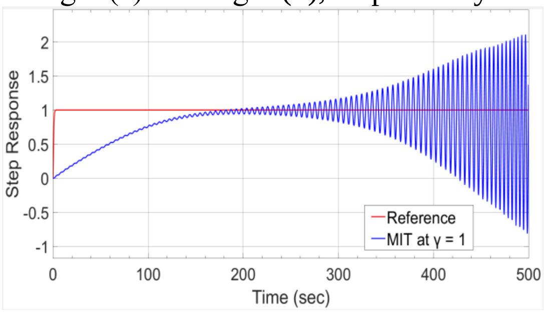


Fig. 4: Block diagram of MRAC using modified MIT rule

7. Performance Evaluations and Simulation Results

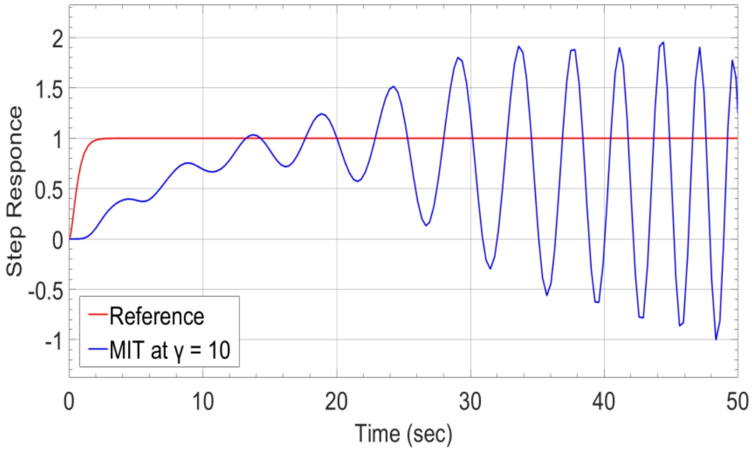
7.1 MIT Rule

In this scheme, a controller for a second-order Inverted Pendulum System has been developed with the MRAC according to the conventional MIT rule. The simulation results for adaptation gain γ = 1 and γ = 10 have been shown in Fig. 5(a) and Fig. 5(b), respectively.



(a)

International Journal of Engineering Technology and Management Sciences
Website: ijetms.in Issue: 1 Volume No.8 January - February – 2024
DOI:10.46647/ijetms.2024.v08i01.031 ISSN: 2581-4621



(b)

Fig.5: Simulation result of MRAC with normal MIT rule for (a) γ =1 and (b) γ = 10

From these results, it has been noticed that with the conventional MIT rule, initially, the system tracks the reference model with oscillations, but after some time, it is unable to stabilize the system and the response goes unbounded. So, to stabilize the system a Fractional-Order MIT rule-based MRAC has been further implemented in the next section.

7.2 Fractional-Order MIT Rule

Implementing the FOMIT rule with fractional orders smaller than one and numerous adaptive gain values has allowed us to examine the effects of adding an extra degree of freedom, alpha. The behavior of the step response of the IP using the FOMIT rule with $\alpha = 0.5$ and multiple γ values has been shown in Fig. 6 and Fig. 7. The FOMIT has been designed to stabilize the IP system. From these results, it has been analyzed that the system response is very sluggish for lower values of adaptation gain. As the adaptation gain increases, the response becomes fast, but after $\gamma = 50$, overshoot is also introduced by analyzing Table 2, it may be observed that the best adaptation gain for FOMIT is 50. Fig. 8 shows the step response of FOMIT at $\gamma = 50$.

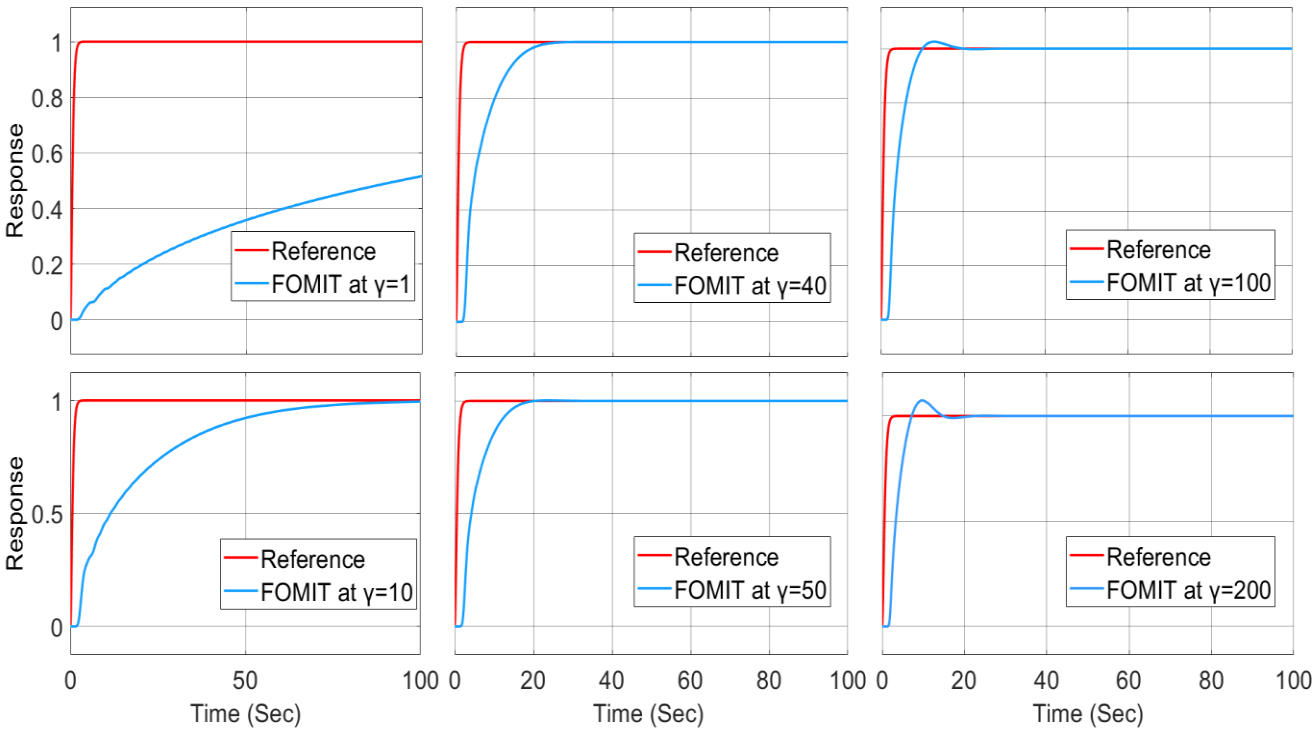


Fig.6: Simulation results of MRAC with FOMIT rule for γ = 1, 10, 40, 50,100 and 200

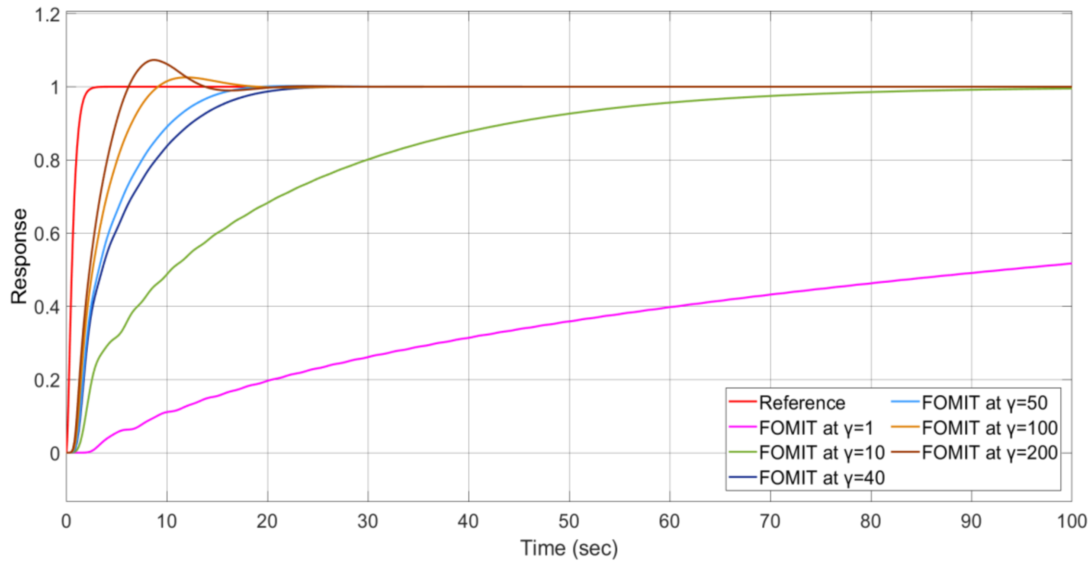


Fig.7: Simulation results of MRAC with FOMIT rule for γ = 1, 10, 40, 50,100 and 200 at the same axis

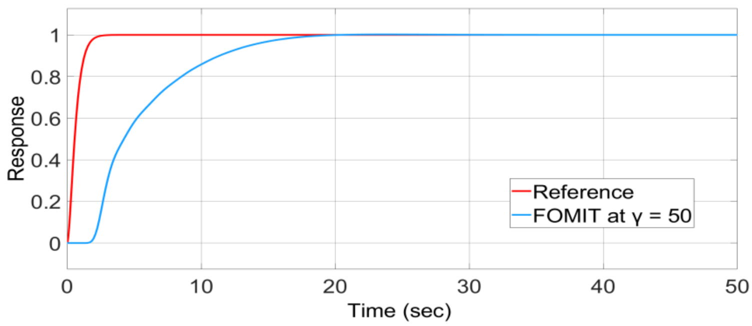


Fig.8: Simulation result of MRAC with FOMIT rule for γ = 50

Now from Table 2, it has been analyzed that with the variation of α from 0.5 to 0.9 and the γ from 1 to 200, the reference model has been tracked well with the desired outcome. With each combination of α and γ, the performance specifications rise time (Tr), settling time (Ts), and peak overshoot (Mp) have been evaluated and shown in Table 2.

Table 2: Performance Indices for Plant Using FOMIT Rule with Deferent Values of α and γ

Alpha (α)	Gamma (γ)	Tr (sec)	Ts (sec)	% Mp
0.5	1	37.9734	48.3582	0
	10	31.4159	45.8398	0
	40	11.0021	19.6969	0.0353
	50	9.0060	16.0973	0.2121
	100	5.3048	14.8397	2.5474
	200	3.8973	13.3131	7.3051
0.9	1	75.8910	96.3745	0
	10	41.4226	71.1113	0
	40	11.0111	19.4209	0.0535
	50	9.0259	15.8712	0.2727
	100	5.3269	14.9234	2.7375
	200	3.8889	13.1330	7.5525

7.3 Modified-MIT Rule

As observed from Fig. 8, the FOMIT is found to stabilize the IPS but responds very slowly with some delay too. The modified MIT rule has been developed to deal with these issues. Here, we have combined the control law of MRAC using the MIT rule with PID control law such that the system

International Journal of Engineering Technology and Management Sciences
Website: ijetms.in Issue: 1 Volume No.8 January - February – 2024
DOI:10.46647/ijetms.2024.v08i01.031 ISSN: 2581-4621

became stable and tracked the desired response. The gains of PID control law (Proportional, Integral, and Derivative) have been fine-tuned using the MIT rule's adaptation parameters.

Ziegler-Nichols PID tuning cannot be employed since the plant model is open-loop unstable. The SISO design toolbox in MATLAB has been used to tune the PID controller based on a reliable response time tuning algorithm. Gains in the PID controller are calculated as follows: Kp = 100, Ki = 10 and Kd = 20.

The behavior of the step response of the IP using the modified MIT rule for a range of adaptation gain from 0.01 to 2 has been shown in Fig. 9. A comparative analysis of the step response of the modified MIT rule-based MRAC scheme has shown in Fig. 10 for multiple values of adaptation gain. From these results, it has been found that for lower values of adaptation gain, the system response is very sluggish. As the adaptation gain increases, the response becomes fast, but after $\gamma = 1$, some overshoot is introduced. By analyzing these results and Table 3, it may observe that the best adaptation gain for the modified MIT rule is 1. The step response of the modified MIT rule at $\gamma = 1$ is shown in Fig. 11.

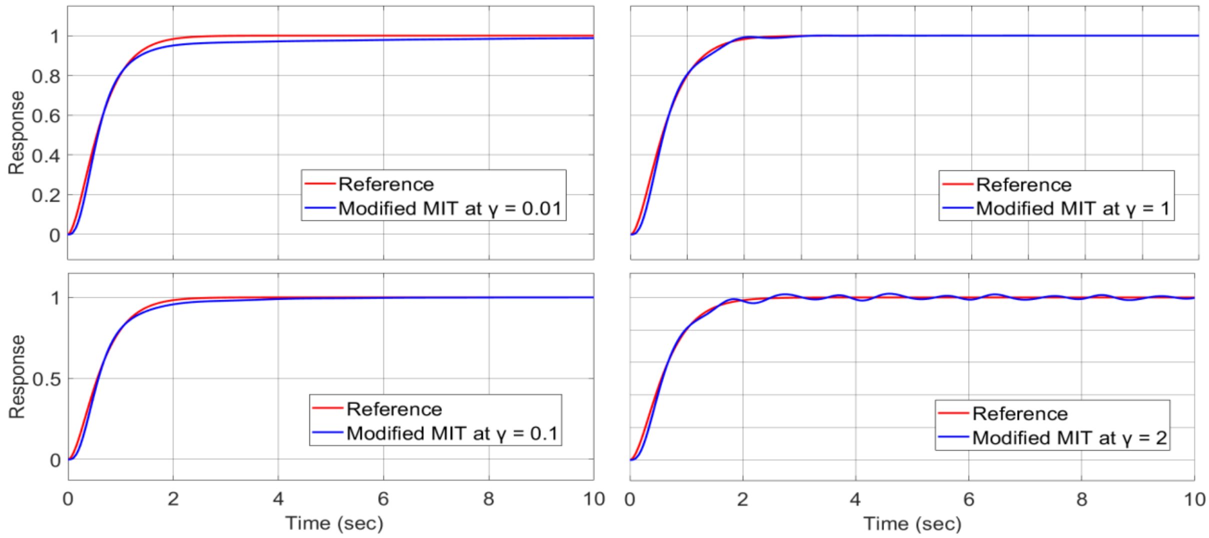


Fig.9: Simulation results of MRAC with modified MIT rule for γ = 0.01, 0.1, 1 and 2

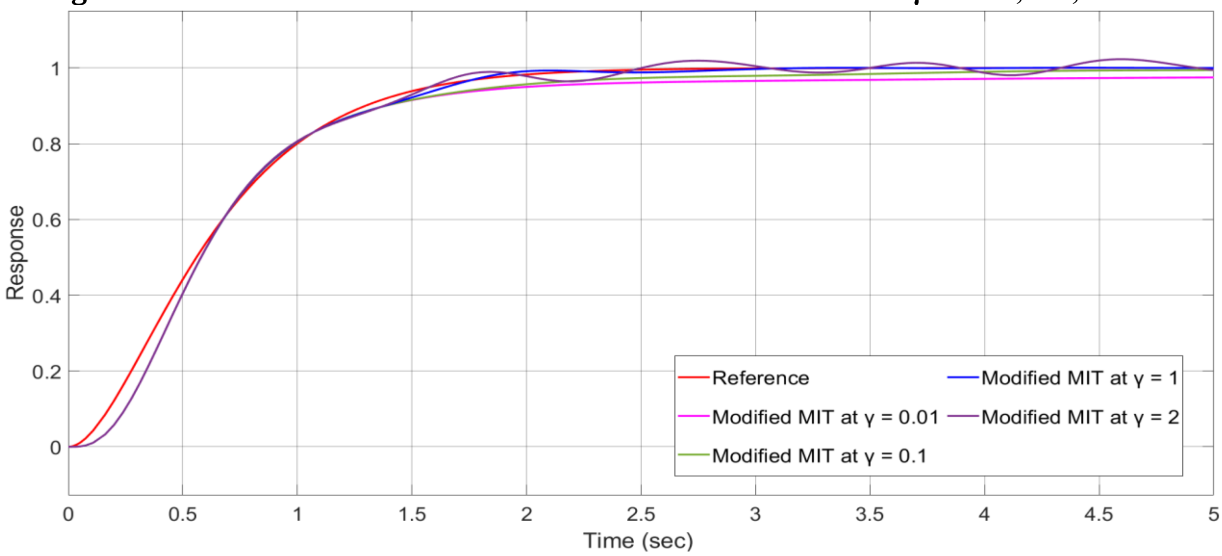


Fig.10: Simulation results of MRAC with modified MIT rule for γ = 0.01, 0.1, 1 and 2 at the same axis

International Journal of Engineering Technology and Management Sciences
Website: ijetms.in Issue: 1 Volume No.8 January - February – 2024
DOI:10.46647/ijetms.2024.v08i01.031 ISSN: 2581-4621

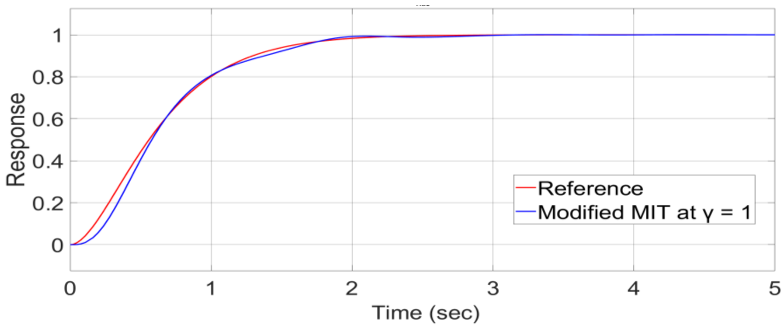


Fig.11: Simulation result of MRAC with modified MIT rule for γ = 1

From Table 3, it has been analyzed that by varying the adaptation gain from 0.01 to 2, the reference model has been tracked well with desired performance with the value of γ = 1. With the adaptation gain from 0.01 to 2, the performance specifications rise time (Tr), settling time (Ts), and peak overshoot (Mp) have been evaluated and shown in Table 3.

Table 3: Performance Indices for Plant Using modified-MIT Rule with Deferent Values of γ

Gamma (γ)	Tr (sec)	Ts (sec)	% Mp
0.01	1.1372	6.5158	5.6317e-04
0.1	1.1363	3.0995	5.5247e-04
1	1.1320	1.8503	0.0508
2	1.1293	4.6928	2.3003

7.4 Comparative Analysis of Results and Discussions

The comparison between step response of normal MIT, FOMIT, and modified MIT rule-based MRAC has been shown in Fig. 12. Table 4 shows the comparative analysis of the responses of the designed controllers w.r.t. rise time (Tr), settling time (Ts) and peak overshoot (Mp).

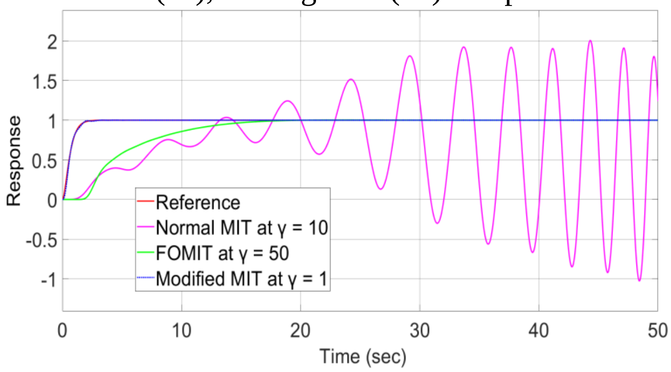


Fig.12: Simulation results of MRAC with MIT rule, FOMIT and modified for rule γ = 10, 50 and 1, respectively

MRAC using the normal MIT rule for IPS was to track the reference model with oscillations initially and after some time, it could not stabilize the system and the response went unbounded. Therefore, the FOMIT rule-based MRAC scheme has been employed to stabilize and study the performance of IPS with one extra degree of freedom alpha. The behavior of the step response of the IP with the FOMIT rule for a range of α and γ from 0.5 to 0.9 and from 1 to 200, respectively, have been analyzed. The FOMIT rule was found to stabilize the IPS and gave the best result at γ = 50, but the response is very sluggish with some delay. To overcome these problems, the modified MIT rule-based MRAC has been designed. The behavior of the step response of IPS with the modified MIT rule for a range of adaptation gain from 0.01 to 2 has been analyzed and it has been found that IPS has stabilized very fast at low adaptation gain compared to FOMIT. From Table 4 it has been concluded that modified MIT rule-based MRAC shows the best response with respect to the performance specifications considered.

Table 4: The comparative analysis of performance indices for the plant using the best result given by all the three mentioned rule

International Journal of Engineering Technology and Management Sciences
Website: ijetms.in Issue: 1 Volume No.8 January - February – 2024
DOI:10.46647/ijetms.2024.v08i01.031 ISSN: 2581-4621

Controller	Gamma (γ)	Tr (sec)	Ts (sec)	% Mp
Normal MIT	-	-	-	-
FOMIT	50	9.0060	16.0973	0.2121
Modified MIT	1	1.1320	1.8503	0.0508

8. Conclusions

In this paper, the traditional MIT, FOMIT and Modified MIT rule-based MRAC have been designed and simulated in MATLAB/Simulink for inverted pendulum system. The performance of normal and fractional order MIT rule-based MRAC systems has been compared to that of the modified systems, which is the MIT technique superimposed with a PID controller in their design. The traditional, fractional order and modified MRAC systems have demonstrated their adaptability and robustness in controlling IPS under various conditions. The adaptability gain parameter, a crucial component of these systems, has been shown to play a significant role in determining their performance. After carefully tuning this gain, we have achieved desirable control performance, with considerations for stability and tracking convergence. Our investigation has highlighted the importance of a balanced adaptation gain. Too high gain can lead to overshooting and instability, while too low gain may result in sluggish responses and poor tracking performance. Achieving the right balance is a critical aspect of the successful implementation of these adaptive control techniques. Through a rigorous examination of these adaptive control strategies, several key findings and insights have emerged. A significant contribution of this paper lies in the comparative stability and adaptation gain analysis of the controller designs. This comparative aspect is crucial for engineers and researchers seeking the most effective control strategy for similar dynamic systems.

The conventional MRAC is suitable only for a few lower adaptation gain values. It has been observed with the conventional MRAC, that the IPS has given very poor results with very high oscillations and reduced tracking performance. To overcome these problems FOMIT rule-based MRAC has been designed. As a result of the fractional order MRAC, the IPS has been stabilized and gave the best result at $\gamma = 50$, but the response is very sluggish with some delay. The behavior of the step response of IPS with the modified MIT rule has been analyzed and it has been found that IPS has stabilized very fast at low adaptation gain compared to FOMIT. Hence, it has been concluded that MRAC using the normal MIT rule is unable to control IPS, and FOMIT rule-based MRAC method needs higher values of adaptation gains, while modified MIT rule-based MRAC shows the desired response at reduced adaptation gain value.

References

1. Verma N and Valluru SK. "Comparative Study of Parametric Disturbances Effect on Cart-Inverted Pendulum System Stabilization". In Modern Electronics Devices and Communication Systems: Select Proceedings of MEDCOM 2021, 2023, pp. 217-230. Singapore: Springer Nature Singapore.
2. Yang Y, Zhang HH, and Voyles RM. "Optimal fractional-order proportional–integral–derivative control enabling full actuation of decomposed rotary inverted pendulum system". Transactions of the Institute of Measurement and Control 2023; 45(10): 4613–4627.
3. M. Bugeja. September. "Nonlinear swing-up and stabilizing control of an inverted pendulum system". In The IEEE Region 8 EUROCON, 2003; 2: pp. 437-441.
4. Assefa A. "Reduced Order Observer Based Pole Placement Design for Inverted Pendulum on Cart". Journal of Informatics Electrical and Electronics Engineering (JIEEE) 2022; 3(2): 1-11.
5. Panjwani B, Kumar V, Yadav J and Mohan V. August. "Optimum LQR Controller for Inverted Pendulum Using Whale Optimization Algorithm". International Conference on Signals, Machines, and Automation 2022, pp. 307-315. Singapore: Springer Nature Singapore.
6. Chacko SJ and Abraham RJ. "On LQR controller design for an inverted pendulum stabilization". International Journal of Dynamics and Control 2022; 11(4): 1-9, 2022.

DOI:10.46647/ijetms.2024.v08i01.031 ISSN: 2581-4621

Wang JJ. Simulation studies of inverted pendulum based on PID controllers. Simulation Modelling Practice and Theory, 2011; 19(1): 440-449.

7. Poonam Jagannath Shinde and Madhumita Chatterjee. "A Novel Approach for Classification and Detection of DOS Attacks." In International Conference on Smart City and Emerging Technology (ICSCET), pp. 1109-8537. IEEE, 2018.

8. Mahmoud MS and Nasir MT. "Robust control design of wheeled inverted pendulum assistant robot". IEEE/CAA Journal of automatica sinica, 2017; 4(4): 628-638.

9. Zhong W and Röck H. "Energy and Passivity Based Control of the Double Inverted Pendulum". In: Proc. of 2001 IEEE Joint International Conference on Control Applications & International Symposium on Intelligent Control, Mexico City, Mexico, 5-7 September 2001, pp. 896-901.

10. Kurdekar V and Borkar S. "Inverted pendulum control: A brief overview". International Journal of Modern Engineering Research 2013; 3(5): 2924-2927.

11. Mukherjee D, Kundu P and Ghosh A. "A Stability Analysis of Inverted Pendulum System Using Fractional-Order MIT Rule of MARC Controller". In Information and Decision Sciences: Proceedings of the 6th International Conference on FICTA 2018, pp. 159-167.

12. Král L and Šimandl M. "Functional adaptive control for multi-input multi-output systems". In 17th International Federation of Automatic Control World Congress, Seoul, Korea, 6-11 July 2008, 41(2): 11263-11268.

13. Dinakin D and Oluseyi P. "Fuzzy-Optimized model reference adaptive control of interacting and noninteracting processes based on MIT and Lyapunov rules". Turkish Journal of Engineering 2021; 5(4): 141-153.

14. Fan JC, and Kobayashi T. "A simple adaptive pi controller for linear systems with constant disturbances". IEEE transactions on automatic control 1998; 43(5): 733-736.

15. Rothe J, Zevering J, Strohmeier M and Montenegro S. "A modified model reference adaptive controller (M-MRAC) using an updated MIT-rule for the altitude of a UAV". Electronics 2020; 9(7): p.1104.

16. Abedini M. "Model Reference Adaptive Control in Fractional Order Systems using Discrete time Approximation Methods". Elsevier 2015; 25: 1–3.

17. Loverro A. "Fractional Calculas: History, Definitions and Applications for the Engineers". Department of Aerospace and Mechanical Engineering, USA, 2004, 11-28.