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Mathematical Beauty in the Taj Mahal

Prashant K Patil[1]
[1] *Assistant Professor, Mathematics, Arts, Science & Commerce College, Mokhada, Palghar, Maharashtra*
Corresponding Author Orcid ID: https://orcid.org/0009-0009-8297-8566

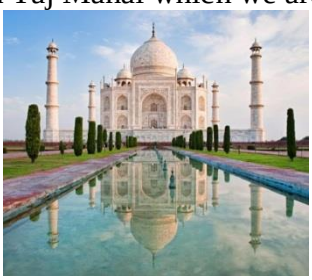
ABSTRACT
Mathematics exists everywhere in nature & around that teaches us how to live life from long time. Human had constructed some of the mysterious and beautiful monuments with the help of Mathematics & Mother Nature which give best example of architecture and engineering. The Taj Mahal is one of the greatest role model of all architectures. The term Golden Ratio plays vital role in quite a few places like nature, sculptures, monuments, architectural design & even in human body which is generally denoted by *phi*(φ) & has value 1.6180339887(approximately)[6]. As it has unique & mystique properties, not only many researchers & mathematicians but also architects, artist & designers have studied, documented & used it. Golden Section is another meaning of Golden Ratio. Golden Ratio can also be seen in the Taj Mahal.

This motivates to make this article to enlighten and analyse the mathematical concepts in the Taj Mahal which is one of the best architectural masterpieces in the world and is great example of architecture based on Mathematics.

Keywords— Line Symmetry, Reflection, Geometry, Golden Ratio, Golden Rectangle, Golden Spiral, Fibonacci sequence, Shape.

1. Introduction

Among the Seven Wonders of the World, the Taj Mahal is one of the elegant & mysterious monuments constructed with the help of mostly translucent white marble by Mughals. On the bank of the Yamuna river in the Agra city (India), a marble mausoleum, "TAJ MAHAL" is situated which is the Paradise on the Earth & sensational architecture was constructed between 1631 to 1648 by Mughal emperor Shah Jahan. Although it only takes up a section of the complex, the mausoleum is the most recognizable aspect of the Taj Mahal The protected complex covers slightly under 42 acres, whereas the overall complex spans 66.62 acres (0.27 square kilometers) (0.169 square km) [11]. This is not only to place the tomb of his favorite wife Mumtaz Mahal (d. 1631) but also to testify to the power and glory of his empire [8]. Taj Mahal is one of the best examples of Line Symmetry as well as geometry. Symmetries can be seen not only in exterior or interior but in the drainage system also. So there is a beauty of Mathematics in Taj Mahal which we are going to look at.



{Fig 1:- Photo Source: http://www.pratirodh.com/news/mud-packs-for-taj-mahal-ahead-of-trumps-visit/}

2. Experimental Methods or Methodology

The researcher has adopted the theoretical content analysis and descriptive method for this research paper. Through identifying the different geometrical patterns, mathematical dimension of golden ratio and its geometric shapes, architecture design & dimension, by analysing historical

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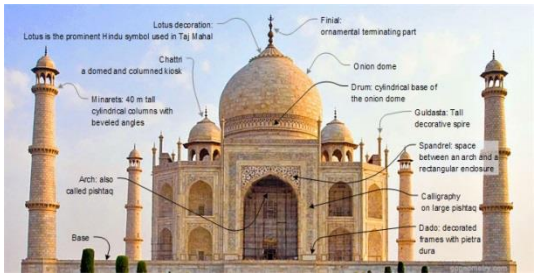
architectural models that has different dimensions of golden ratio, in order to get out with theoretical findings that define golden ratio as a basis for architectural development of Taj Mahal.

The most significant monument in India according to the Archaeological Survey of India is the Taj Mahal. In essence, it is a double-domical structure held up by eight enormous columns, with arches connecting the columns. The entire building is supported by a 33.5 m thick raft that is approximately 18.3 m thick above ground. The raft is supported by a thin 9.1 m-thick layer of sand, which is followed by a 63.4 m-thick layer of clayey strata that is topped by rock. Brick masonry, comprising 3.8 cm thick fire-burnt clay bricks and mortar joints varied in thickness up to two inches, is used to construct the entire superstructure and the raft. The Taj Mahal is constructed on a square, solid platform that is 33.5 metres thick, 98.2 metres square in plan, and 15.2 metres thick, with the latter portion buried beneath the earth.. There is a 9.1 m deep layer of sand on this platform, and 63.6 m thick clay underneath.. Natural rock is located at a depth of 88.0 m below the ground level [13].

Right from the beginning i.e. main gate of the Taj Mahal up till main dome of the Taj Mahal we can see the presence of Mathematics at every step. We are going to pay our attention on some of the terms of Mathematics viz. Line Symmetry, Reflection, Geometry, Golden Ratio, several shapes of monument.



{Fig 2:- Photo Source:
http://www.gogeometry.com/wonder_world/taj_mahal_geometry_architecture.html}

The Main Gate (The Great Gate):

The Great Gate lies in the middle of the forecourt and is on the southern wall of the symmetrical funerary garden, which is divided into four sections. [11]. The main gate provide nice Optical illusion to visitors – as one approaches closer to the Taj Mahal from main gate, it appears to be moving further away from you and seems to get smaller & smaller & grow to bigger when we walk away.

(Fig 3a) (Fig 3b)

{Fig 3(a):- Photo Source: https://whereisdayna.com/2016/03/25/taj-mahal-a-closer-look/
{Fig 3(b)Photo Source: https://mapio.net/pic/p-70117182/}

Line Symmetry: Reflection in water

If we look closely at the Taj Mahal from a particular distance, we may see a stunning example of line symmetry with two lines (like XY axes), one running vertically downhill from the centre of the Taj and the other along the water's edge, which shows us the reflection of the prayer towers in the water.

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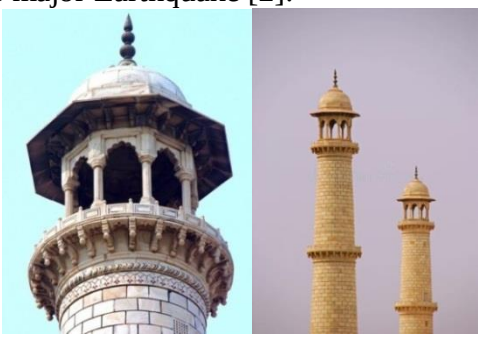
{Fig 4:- Photo Source: https://hdwallpaperim.com/reflection-taj-mahal/}

Minarets:

The Taj's four minarets are each 41.1 meters tall [3], with an octagonal base and a thin cylindrical body encircled by an eight-column chhatri. When we observe sharply, we'll find that the four minarets are slightly tilting outwards, for arriving visitors and to protect main Dome and meanwhile the Tomb from any natural calamities like major Earthquake [2].

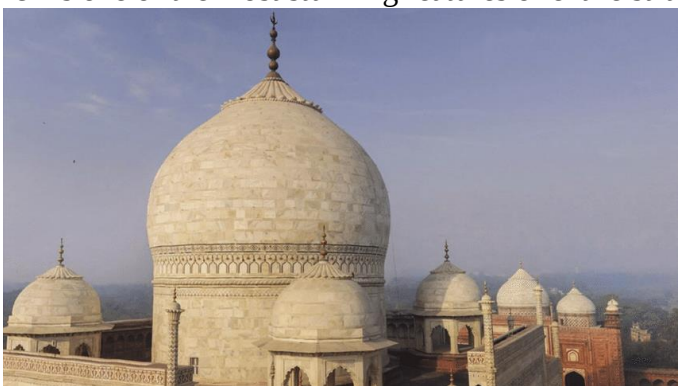


(Fig 4a) (Fig 4b)

{Fig 4a:- Photo Source: (https://www.pinterest.com/pin/276197389624120806/
https://www.dreamstime.com/royalty-free-stock-
Fig 4a:- photography-minaret-taj-mahal-two-minarets-standing-parallel-image31207657}

The Dome:

The Shape of the dome is remarkable; it is Onion in shape, also sometimes referred as "bulbous dome". It is mounted on a 17m & 68cm diameter circle and measures 35m high. It places on 7m heighted drum & is flanked by four smaller domes. At 115 ft. the marble cylindrical dome is topped with design of lotus, which is one of the most stunning features of entire structure [2].



{Fig 5:- Photo Source: https://www.researchgate.net/figure/Photograph-of-the-central-dome-of-the-Taj-Mahal-showing-the-supporting-drum_fig1_299356450/}

The Tomb:

The tombs of Shah Jahan and Mumtaz Mahal are covered with white marble curtains of different designs and semi-precious stones [10]. The room where Tombs are situated inside the Taj Mahal is Octagonal in shape [3].

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(Fig 6a) (Fig 6b)

{Fig 6(a):- Photo Source:
http://mariam-uz-zamani.blogspot.com/2015/02/the-taj-mahal-labour-of-love.html#.YrVAOv1BzIU}
{Fig 6(b):- Photo Source:
http://www.anindianmuslim.com/2013/03/inside-taj-mahal-seeing-replica-of-shah.html}

Floors and Drainage:

Floors which were made by marbles are Diamond in shape & are arranged in symmetry in such a way that it make star in between.

{Fig 7:- Photo Source: https://www.pinterest.com/pin/679691768748252559/}

Also tiling pattern consist of extended hexagonal in shape and arranged in such a way that it make four pointed star in between as shown in figure.

{Fig 8:- Photo Source: http://mathtourist.blogspot.com/2010/09/tilings-at-taj-mahal.html}

In another place of Taj, the tiling pattern mixes a six-pointed star in the middle of a normal hexagon.

(Fig 9a) (Fig 9b)

{Fig 9a:- Photo Source:
http://mathtourist.blogspot.com/2010/09/tilings-at-taj-mahal.html
Fig 9b:- https://www.dreamstime.com/detail-geometric-floor-tiling-outside-taj-mahal-stars-hexagonal-shapes-warm-pink-colors-seen-looking-down-image164287286}

Drainage system can be seen in the Taj Mahal. In some of the stones, drainage holes have a striking hexagonal pattern.[4]

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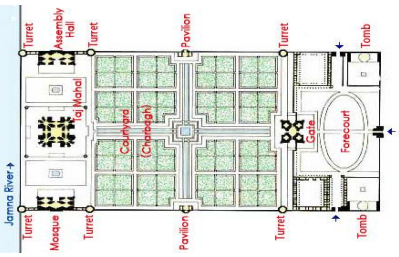
{Fig 10:- Photo Source: http://mathtourist.blogspot.com/2010/09/tilings-at-taj-mahal.html}

The Garden:

The garden which is also called *Charbagh*; situated in front of Taj Mahal is also an example of mathematical symmetries. Tomb garden and its forecourt are fully preserved; as per research & measurement of Ebba Koch and Richard André Barraud which is brought into light again by Sanjay Surya, he measured it as 561.20 x 300.84(300) m [1][8]. Walkway stones are laid in pattern that combines squares & elongated hexagons to create regular octagons. Our eyes are drawn in parallel lines to the entrance of the tombs by the rectangular reflecting pool, which mirrors the pools in each of the four gardens. The intersecting perpendicular lines that create right angles in each of the four sections which are further subdivided into another four squares. The entrances have semi-octagonal angles and are rectangular in shape. [3]

(Fig 11a)

(Fig 11b)



(Fig 11c)

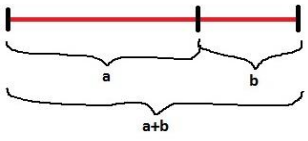
{Photo Source:
Fig 11a:- https://travel.sygic.com/en/poi/taj-mahal-garden-poi:5159456
Fig 11a:- https://www.wonders-of-the-world.net/Taj-Mahal/Gardens-of-the-Taj-Mahal.php
Fig 11c:- http://www.kamit.jp/02_unesco/15_tajmahal/taj_eng.htm}

Here, we have seen that not only exterior and interior but garden and even drainage system is based on geometrical concepts.

Golden Ratio, Golden Rectangle and Fibonacci sequence in Taj Mahal:

There are numerous definitions of the golden ration. Euclid; 'Father of Geometry', who was the first one who discussed about the golden ratio definition and said that "A straight line to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less". Golden ratio is a ratio between sections or dimension of one element. According to Edmund Harriss (Harriss, 2015) if there is a pure rectangle with subdividing for as long as to the smaller rectangles into a square and even a smaller golden rectangle between two sections of it [14].

Golden ratio is a magical number which is irrational number [7], denoted by φ and has approximate value $\varphi = (1+\sqrt{5})/2 = 1.6180339887498948482$…..with unending decimal places [5]



$$\frac{a}{b} = \frac{a+b}{a} = \phi = 1.6180339887$$

(Figure 12)

It is stated that two amounts a and b are in the golden ratio (See Figure 12) if

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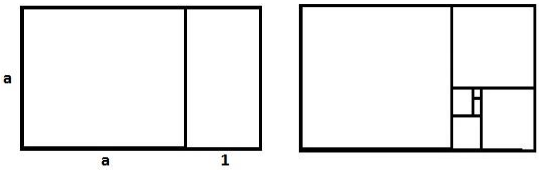
$\frac{a+b}{a} = \frac{a}{b} = \varphi$ (say). ……...(A)

After simplifying given equation (A), we get quadratic equation as

$\varphi^2 - \varphi - 1 = 0$ ……..(B)

On solving equation (B) by quadratic formula, we get two solutions as $\varphi = \frac{1\pm\sqrt{5}}{2}$. Since φ is considered for positive quantities, so $\varphi = \frac{1+\sqrt{5}}{2} = 1.6180339887$

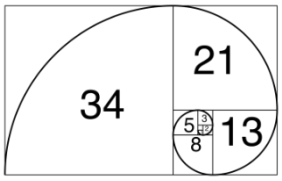
This number can also be found out as the solution to the problem of subdividing rectangle into square of area a^2 & remaining, smaller rectangle that is similar to original large rectangle. (See Figures below)



(Figure 13a) (Figure 13b)

Figure 13b shows entire rectangle is subdivided into small squares & smaller rectangles (with same aspect ratio) that are similar to original rectangle & we call it as Golden rectangle. A rectangle whose dimensions are in Golden ratio is called Golden Rectangle.

The given problem is given by the relation $\frac{a+1}{a} = \frac{a}{1}$, which results in equation $a^2 - a - 1 = 0$. After solving this equation same as earlier solved above we get positive solution as $a = \varphi = (1+\sqrt{5})/2 = 1.6180339887$(approx.) [6] which is an irrational number. There is a relation between Golden Mean & Fibonacci sequence [7]. Let $(x_n)_{n\in\mathbb{N}}$ be sequence of positive numbers which is defined by recursive formula $x_{n+2} = x_{n+1} + x_n$, with $x_1 = x_2 = 1$. This gives us {1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, ...}. This sequence is calculated by simply adding previous number in present number to get next number.

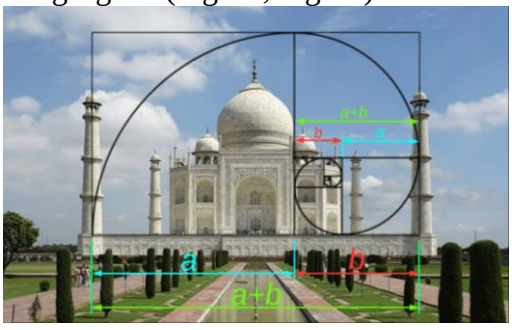


{Fig 14:- Photo Source: https://www.studypug.com/blog/beauty-and-the-beast/}

We can obtain rational approximation to Golden ratio φ by considering ratios of consecutive numbers in the Fibonacci sequence i.e. 8/5 = 1.60, 13/8 = 1.625, 21/ 13 = 1.615,… Hence, the ratio of successive terms in the Fibonacci sequence tends to the Golden ratio φ viz.

$x_{n+1} / x_n \rightarrow 1.6180339887\ldots$ This result is given by the great astronomer Johannes Kepler [9].

In the Taj Mahal, the Golden Ratio is seen clear in terms of ratio of the height to the width of the Arches [12]. The Golden rectangles & Golden Spiral can be seen within the Main building of Taj Mahal [5][1] as shown in following figures(Fig-15, Fig-16).



{Fig 15: - Photo Source: https://tr.pinterest.com/pin/241857442473367619/?amp_client_id=CLIENT_ID(_)&mweb_unauth_id={{default.session}}&amp_url=https%3A%2F%2Ftr.pinterest.com%2Famp%2Fpin%2F241857442473367619%2F&from_amp_pin_page=true}

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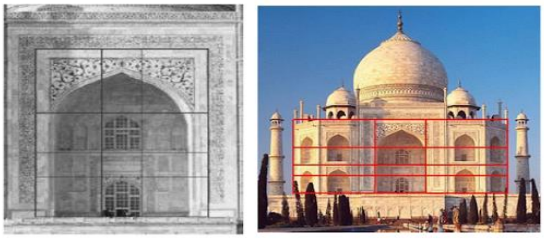

{Fig 16:- Photo Source: article.sapub.org/image/10.5923.j.arts.20201001.01_004.gif}

3. Results and Discussion

Mathematics involves not just numbers; but it also involves relationships and ratios between the numbers. This research tries to find the earliest examples of the most prominent types of geometrical pattern of Mughal architecture, especially of Taj Mahal. This involves study & analysis of concepts of mathematics like Geometry, Golden ratio, Line symmetry, Shapes & sizes. The Taj Mahal keeps before us not only Mughal culture in decoration but also a wonderful example of Engineering & Architecture. This study highlights the significance of mathematics to drainage systems in addition to the Taj Mahal's fundamental dome design. Hence, Society benefits from mathematics both directly and indirectly in a variety of ways, including design and building construction.

CONCLUSION

The Taj Mahal is one of the best examples of Architectural Masterpiece constructed using mathematics which includes Geometry, Shapes, symmetries, & Golden proportion. In today's scenario, there is a lack of use of Golden Ratio or not given much importance to Golden Ratio in the architecture or designing building. This study looks into how fundamental ideas and concepts in architecture are connected to mathematics in general. It is evident that mathematics has a unique aesthetic when applied more correctly in building design. For architects, the Golden Ratio is a universal concept that is used in a variety of contexts, including furniture, low-rise buildings, and skyscrapers. The evidence and studies demonstrate how the golden ratio affects architectural design. In actuality, the golden ratio is more than just an equation and some rules; it can also be used in intricate architecture at various scales. If we use Golden ratio more frequently then we can create more improved structures by design and stability. Keeping this in mind, in the future we can keep building such kind of constructions, architecture or monument using Golden Proportion that will last for long years.

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