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Application of Non-Linear Programming to Portfolio Management on some Insurance Companies (AIICO, LINKAGE, NIGER, Mutual benefit and LASACO) using Return on Asset

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***Abstract*— This study investigate non-linear programming problem that is, quadratic programming and its application to portfolio management. The data of return on asset of five different insurance companies namely: AIICO, LINKAGE, NIGER, MUTUAL BENEFIT, and LASACO insurance company was collected and a model was fixed. These data were analyzed using quadratic programming in conjunction with LINGO software. The result of the analyzed data revealed that the allocation of fund for each insurance companies should be done with the same percent for LINKAGE, NIGER, MUTUAL BENEFIT and other percent to AIICO insurance company respectively with increment of 24% on return.**

***Keywords*— Return on assets, insurance companies, portfolio management and model.**

I. INTRODUCTION

Portfolio models are concerned with investment where there are typically two criteria: expected return and risk. The investor wants the former to be high and the latter to be low. There is a variety of measures of risk. The most popular measure of risk has been variance in return introduced by [12]. Quadratic programming is computationally appealing because the algorithms for linear programs can be applied to quadratic programming with only modest modifications. Loosely speaking, the reason only modest modification is required is the first derivative of a quadratic function is a linear function. Because LINGO has a general nonlinear solver, the limitation to quadratic functions is helpful, but not crucial.

Quadratic programming is a unique case of mathematical optimization problem. It is the problem of maximizing or minimizing a quadratic function where the variables are subject to linear constraints. Quadratic programming is used in portfolio optimization for the formulation of the mean-variance optimization of investments judgments under uncertainty.

Portfolio management, for an investor that has different assets to trade in, is choosing optimal investments, that is, how many shares of an asset should he/she buy and hold for him/her to maximize some criteria depending on his/her total wealth and/or consumption.

Return on assets (ROA) is a financial ratio that shows the percentage of profit a company earns in relation to its overall resources. It is commonly defined as net income divided by total assets. Net income is derived from the income statement of the company and it is the profit after taxes. The assets are read from the balance sheet and include cash and cash equivalent items such as receivables, inventories, land, capital equipment as depreciated, and the value of intellectual property such as patents.

Portfolio is a collection or an aggregation of investments tools such as stocks, shares, mutual funds, bonds, cash etc [4]. It also indicate that the decision of future yet unknown is premise on the information gotten from the past.

[5] Used return on invested capital to investigate how much Dangote can invest on three of his subsidiaries Viz. Dangote Cement, Dangote Sugar refinery and Dangote Flour given an amount available to him.

[1] Used the quadratic approach to select the optimum portfolio of the Malaysian stock exchange and her framework deals with ten biggest firms posted on the stock exchange during 2014. The result shows that the optimum portfolio includes 22 % of Axiata Group shares, 11% of Genting shares, 30 % of Petronas Chemicals shares, 1% of Sime Darbi shares and 36 % of Tenaga Nasional shares.

[6] In his thesis used dividend payout ratio as a determinant to investigate how to make selection of Bank shares in three different Banks, which are Zenith Bank, Guaranty Trust Bank plc, and First Bank plc.

However, this study is working on five (5) different insurance companies which are AIICO Insurance Company, Linkage Insurance Company, LASACO Insurance Company, Niger Insurance Company and Mutual Benefit Insurance Company to investigate the percentage to invest on each company’s return on asset.

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I. DATA PRESENTATION

For the purpose of this work, abstraction from established published sources was used to collect data. The data used was already in existence but were extracted for the purpose of this work and explained briefly below.

The table1: below shows the Percentage Return on Asset invested on Linkage insurance company, Mutual Benefit insurance company, Niger insurance company, AIICO insurance company and LASACO insurance company.

Insurance company	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
LINKAGE	2.8	3.1	3.9	4.2	4.4	3.7	3.9	3.9	3.9	3.3
MUTUAL BENEFIT	2.5	2.0	3.1	3.5	3.0	3.0	3.6	3.3	2.8	2.7
NIGER	0.2	1.2	1.8	1.1	1.4	2.4	1.5	2.1	2.1	2.3
AIICO	2.2	3.7	2.6	2.2	0.9	2.6	0.7	1.3	2.1	1.1
LASACO	2.6	3.1	3.6	3.1	2.4	2.0	1.8	2.3	3.1	1.6

Source: Annual financial record on Return on asset of five (5) selected insurance companies.

II. DATA ANALYSIS

An investor has fixed sum of money say K, to invest in five (5) insurance companies namely; Linkage, Mutual Benefit, Niger, AIICO and LASACO.

The Portfolio problem is to determine how much money the investor should allocate to each insurance company so that total expected return is greater than or equal to some lowest acceptable amount say T, and so that the total variance of future payment is minimized.

Let X_1, X_2, X_3, X_4, X_5 designate the amount of money to be

allocated to Linkage insurance company, Mutual Benefit insurance company, Niger insurance company, AIICO insurance company, and LASACO insurance company respectively and let X_{is} denote the return per naira invested from invested from the investment **i (i = 1, 2, 3, 4, 5)** during the S period of time in the past **(S = 1, 2, 3, … 10).** If the past history on return on asset is indicative of future performance, the expected future return per Naira from investment 1, 2,3,4,5 is

$$E_i = \frac{\sum_{s=x1}^{10} bis}{10} \quad (1)$$

And the expected return from five investments combines is

$$E = E_1X_1 + E_2X_2 + E_3X_3 + E_4X_4 + E_5X_5 \quad (2)$$

The portfolio problem modeled as quadratic programming is

$$\text{Min } R = A^TCA$$

Subject to: $X_1 + X_2 + X_3 + X_4 + X_5 = N$

$E X_1 + E X_2 + E X_3 + E X_4 + E X_5 \geq K$

$X_1 \geq 0, X_2 \geq 0, X_3 \geq 0, X_4 \geq 0, X_5 \geq 0$, where C is the covariance matrix which is positive semi – definite i.e.

Matrix $C = \begin{pmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \sigma_{13}^2 & \sigma_{14}^2 & \sigma_{15}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \sigma_{23}^2 & \sigma_{24}^2 & \sigma_{25}^2 \\ \sigma_{31}^2 & \sigma_{32}^2 & \sigma_{33}^2 & \sigma_{34}^2 & \sigma_{35}^2 \\ \sigma_{41}^2 & \sigma_{42}^2 & \sigma_{43}^2 & \sigma_{44}^2 & \sigma_{45}^2 \\ \sigma_{51}^2 & \sigma_{52}^2 & \sigma_{53}^2 & \sigma_{54}^2 & \sigma_{55}^2 \end{pmatrix}$, E is the mathematical expectation, A is column Matrix = $\begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix}$

Subject to: $X_1 + X_2 + X_3 + X_4 + X_5 = N$

$E X_1 + E X_2 + E X_3 + E X_4 + E X_5 \geq K$

$X_1 \geq 0, X_2 \geq 0, X_3 \geq 0, X_4 \geq 0, X_5 \geq 0.$

Hence obtain

Min $R = x_1^2\sigma_{11}^2 + x_2^2\sigma_{22}^2 + x_3^2\sigma_{33}^2 + x_4^2\sigma_{44}^2 + x_5^2\sigma_{55}^2 + X_1X_2(\sigma_{21}^2 + \sigma_{12}^2) + X_1X_3(\sigma_{31}^2 + \sigma_{13}^2) + X_1X_4(\sigma_{41}^2 + \sigma_{14}^2) + X_1X_5(\sigma_{51}^2 + \sigma_{15}^2) + X_2X_3(\sigma_{32}^2 + \sigma_{23}^2) + X_2X_4(\sigma_{42}^2 + \sigma_{24}^2) + X_2X_5(\sigma_{52}^2 + \sigma_{25}^2) + X_3X_4(\sigma_{43}^2 + \sigma_{34}^2) + X_3X_5(\sigma_{53}^2 + \sigma_{35}^2) + X_4X_5(\sigma_{54}^2 + \sigma_{45}^2) + (X_1 + X_2 + X_3 + X_4 + X_{5-1})$ UNITY + [1.20 – $(E_{x1}X_1 + E_{x2}X_2 + E_{x3}X_3 + Ex_4X_4 + E_{x4}X_4 + E_{x5}X_5$ RETURN + $(X_1 - 0.25)\ X_1$ FRAC + $(X_2 – 0.25)\ X_2$ FRAC + $(X_3 –0.25)\ X_3$ FRAC + $(X_5 – 0.25)\ X_5$ FRAC

CALCULATED COVARIANCE MATRIX FROM MINITAB

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Matrix

$$C = \begin{pmatrix} 0.25 & 0.18 & 0.12 & -0.20 & 0.03 \\ 0.18 & 0.23 & 0.07 & -0.27 & -0.06 \\ 0.12 & 0.07 & 0.45 & -0.11 & -0.13 \\ -0.20 & -0.27 & -0.11 & 0.87 & 0.40 \\ -0.03 & -0.06 & -0.13 & 0.40 & 0.43 \end{pmatrix}$$

Expected returns of return of asset for each insurance companies are 3.71%, 2.95%, 1.61%, 1.94, 2.56% respectively. The budget constraint investment portfolio optimization problem has five candidate assets (X_1, X_2, X_3, X_4, X_5) for our portfolio.

A. MODEL

Determine what fraction should be devoted (or of the return on asset that the investor should have) to each insurance company, so an expected return of at least 25% (equivalently, a growth factor 1.25) is obtained while minimizing the variance in return and not exceeding a budget constraint.

It also impose a restriction that any given assets can constitute at most 25% of the portfolio. The variance if the entire portfolio is;

$R=0.25X_1^2 + 0.23X_2^2 + 0.45X_3^2 + 0.87X_4^2 + 0.43X_5^2 + 0.18X_1X_2 + 0.12X_1X_3 – 0.20X_1X_4 + 0.03X_1X_5 + 0.07X_2X_3 – 0.27X_2X_4 – 0.06X_2X_5 – 0.11X_3X_4 – 0.13X_3X_5 + 0.40X_4X_5$

Since variance is a measure of risk, we need to minimize, Hence

MIN $R =0.25X_1^2 + 0.23X_2^2 + 0.45X_3^2 + 0.87X_4^2 + 0.43X_5^2 + 0.18X_1X_2 + 0.12X_1X_3 – 0.20X_1X_4 + 0.03X_1X_5 + 0.07X_2X_3 – 0.27X_2X_4 – 0.06X_2X_5 – 0.11X_3X_4 – 0.13X_3X_5 + 0.40X_4X_5$

Subject to:

! It starts with #1.00

$X_1 + X_2 + X_3 + X_4 + X_5 = 1$

! We want to end with at least #1.20

$3.71X_1 + 2.95\ X_2 + 1.61X_3 + 1.94X_4 + 2.56X_5 \geq 1.20$

! No asset may constitute more than 25% of the portfolio

$X_1 < 0.25$

$X_2 < 0.25$

$X_3 < 0.25$

$X_4 < 0.25$

$X_5 < 0.25$

The research employs LINGO software to create the Lagrangian expression. The input procedure for LINGO requires the model be converted to through linear form by writing the first order conditions. To do this we introduce Lagrangian multiplier for each constraint. There are seven (7) constraints, we shall use seven (7) dual variables devoted respectively as UNITY, RETURN, X_1FRAC, X_2FRAC, X_3FRAC, X_4FRAC, X_5FRAC.

The Lagrangian expression corresponding to this model is now

MIN R (X_1, X_2, X_3, X_4, X_5) $=0.25X_1^2 + 0.23X_2^2 + 0.45X_3^2 + 0.87X_4^2 + 0.43X_5^2 + 0.18X_1X_2 + 0.12X_1X_3 – 0.20X_1X_4 + 0.03X_1X_5 + 0.07X_2X_3 – 0.27X_2X_4 – 0.06X_2X_5 – 0.11X_3X_4 – 0.13X_3X_5 + 0.40X_4X_5 + (X_1 + X_2 + X_3 + X_4 + X_5)$ UNITY + [1.20 – ($3.71X_1 + 2.95\ X_2 + 1.61X_3 + 1.94X_4 +$ 2.56X5)RETURN + (X_1- 0.25) X_1 FRAC + (X_2 – 0.25) X_2 FRAC + (X_3 –0. 25) X_3 FRAC + (X_5 – 0.25) X_5 FRAC

Next to compute the first order conditions

$\frac{\partial R}{\partial X1} = 0.5X_1 + 0.18X_2 + 0.12X_3 - 0.20X_4 + 0.03X_5 +$ UNITY – 3.71 RETURN + X_1 FRAC > 0

$\frac{\partial R}{\partial X2} = 0.18X_1 + 0.46X_2 + 0.07X_3 – 0.27X_4 - 0.06X_5 +$ UNITY – 7.95 RETURN+ X_5 FRAC > 0

$\frac{\partial R}{\partial X3} = 0.12X_1 + 0.07X_2 + 0.9X_3 – 0.11X_4 – 0.13X_5 +$ UNITY – 1.61 RETURN + X_3 FRAC > 0

$\frac{\partial R}{\partial X4} = -0.20X_1 – 0.27X_2 – 0.11X_3 + 1.74X_4 + 0.40X_5$ +UNITY – 1.94 RETURN + X_4 FRAC > 0

$\frac{\partial R}{\partial X5} = 0.03X_1 – 0.06X_2 – 0.13X_3 + 0.40X_4 + 0.86X_5 +$ UNITY – 2.56 RETURN + x_5 FRAC > 0

$$\frac{\partial R}{\partial UNITY} = X_1 + X_2 + X_3 + X_4 + X_5 – 1$$

$$\frac{\partial R}{\partial RETURN} = 1.20\ (E_{x1}X_1 + E_{x2}X_2 + E_{x3}X_3 + E_{x4}X_4 + E_{x5}X_5)$$

Adding the real constraints

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$X_1 + X_2 + X_3 + X_4 + X_5 = 1$

$3.71X_1 + 2.95X_2 + 1.61X_3 + 1.94X_4 + 2.56X_5 \geq 1.20$

$X_1 < 0.25$

$X_2 < 0.25$

$X_3 < 0.25$

$X_4 < 0.25$

$X_5 < 0.25$

The final model is

Min $X_1 + X_2 + X_3 + X_4 + X_5$ + UNITY + RETURN + X_1FRAC+ X_2 FRAC + X_3FRAC + X_4FRAC + X_5 FRAC

! First order condition for X_1:

$0.5x_1 + 0.18x_2 + 0.12x_3 – 0.20x_4 + 0.03x_5$ + UNITY – 3.71 RETURN + X_1FRAC >0

! First order condition for X_2:

$0.18X_1 + 0.46X_2 + 0.07X_3 – 0.27X_4 – 0.06X_5$ + UNITY – 2.95 RETURN + X_2 FRAC > 0

! First order condition for X_3:

$0.12X_1 + 0.07X_2 + 0.93X_3 – 0.11X_4 – 0.13X_5$ + UNITY – 1.61 RETURN + X_3 FRAC > 0

! First order condition for X_4:

$-0.20X_1$ - 0.27X2 - 0.11X3 + $1.74X_4 + 040X_5$ + UNITY – 1.94 RETURN + X_4 FRAC > 0

! First order condition for X_5:

$0.03X_1 - 0.06X_2 - 0.13X_3 + 0.40X_4 + 0.86X_5$ + UNITY – 2.56 RETURN + X_5 FRAC > 0

! …………. Start of "real" constraints…………

! Budget Constraint, multiplier is UNITY.

$X_1 + X_2 + X_3 + X_4 + X_5 = 1$

!Growth constraint, multiplier is RETURN:

$3.7X_1 + 2.95X_2 + 1.61X_3 + 1.94X_4 + 2.56X_5 > 1.20$

! Max Fraction 0f X_1, multipliers is X_1 FRAC:

$X_1 < .25$

! Max Fraction 0f X_2, multipliers is X_2 FRAC:

$X_2 < .25$

! Max Fraction 0f X_3, multipliers is X_3 FRAC:

$X_3 < .25$

! Max Fraction 0f X_4, multipliers is X_4 FRAC:

$X_4 < .25$

! Max Fraction 0f X_5, multipliers is X_5 FRAC:

$X_5 < .25$

END

QCP 7

VI. RESULT OF THE MODEL OBTAINED FROM LINDO SOFTWARE

AT 20%
LP OPTIMUM FOUND AT STEP 5
OBJECTIVE FUNCTION VALUE
1) 1.000000

VARIABLE	VALUE	REDUCED COST
X1	0.200000	0.000000
X2	0.200000	0.000000
X3	0.200000	0.000000
X4	0.200000	0.000000
X5	0.200000	0.000000
UNITY	0.000000	1.000000
RETURN	0.000000	1.000000
X1FRAC	0.000000	1.000000
X2FRAC	0.000000	1.000000
X3FRAC	0.000000	1.000000
X4FRAC	0.000000	1.000000
X5FRAC	0.000000	1.000000

NO. ITERATIONS =5

AT 21%

LP OPTIMUM FOUND AT STEP 5

OBJECTIVE FUNCTION VALUE

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1) 1.000000

VARIABLE	VALUE	REDUCED COST
X1	0.210000	0.000000
X2	0.210000	0.000000
X3	0.210000	0.000000
X4	0.160000	0.000000
X5	0.210000	0.000000
UNITY	0.000000	1.000000
RETURN	0.000000	1.000000
X1FRAC	0.000000	1.000000
X2FRAC	0.000000	1.000000
X3FRAC	0.000000	1.000000
X4FRAC	0.000000	1.000000
X5FRAC	0.000000	1.000000

NO. ITERATIONS =5

AT 22%

LP OPTIMUM FOUND AT STEP 5

OBJECTIVE FUNCTION VALUE

1) 1.000000

VARIABLE	VALUE	REDUCED COST
X1	0.220000	0.000000
X2	0.220000	0.000000
X3	0.220000	0.000000
X4	0.120000	0.000000
X5	0.220000	0.000000
UNITY	0.000000	1.000000
RETURN	0.000000	1.000000
X1FRAC	0.000000	1.000000
X2FRAC	0.000000	1.000000
X3FRAC	0.000000	1.000000
X4FRAC	0.000000	1.000000
X5FRAC	0.000000	1.000000

NO. ITERATIONS =5

AT 23 %

LP OPTIMUM FOUND AT STEP 5

OBJECTIVE FUNCTION VALUE

1) 1.000000

VARIABLE	VALUE	REDUCED COST
X1	0.230000	0.000000
X2	0.230000	0.000000
X3	0.230000	0.000000
X4	0.080000	0.000000
X5	0.230000	0.000000
UNITY	0.000000	1.000000
RETURN	0.000000	1.000000
X1FRAC	0.000000	1.000000
X2FRAC	0.000000	1.000000
X3FRAC	0.000000	1.000000
X4FRAC	0.000000	1.000000
X5FRAC	0.000000	1.000000

NO. ITERATIONS =5

AT 24%

LP OPTIMUM FOUND AT STEP 5

OBJECTIVE FUNCTION VALUE

1) 1.000000

VARIABLE	VALUE	REDUCED COST
X1	0.240000	0.000000

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X2	0.240000	0.000000
X3	0.240000	0.000000
X4	0.040000	0.000000
X5	0.240000	0.000000
UNITY	0.000000	1.000000
RETURN	0.000000	1.000000
X1FRAC	0.000000	1.000000
X2FRAC	0.000000	1.000000
X3FRAC	0.000000	1.000000
X4FRAC	0.000000	1.000000
X5FRAC	0.000000	1.000000

NO. ITERATIONS =5

Table2: The summary of the results yield the table below for the purpose of comparison and decisions

T	X1	X2	X3	X4	X5	Variance	LP optimum step
1.20	0.200000	0.200000	0.200000	0.200000	0.200000	1.000000	5
1.21	0.210000	0.210000	0.210000	0.160000	0.210000	1.000000	5
1.22	0.220000	0.220000	0.220000	0.120000	0.220000	1.000000	5
1.23	0.230000	0.230000	0.230000	0.80000	0.230000	1.000000	5
1.24	0.240000	0.240000	0.240000	0.40000	0.240000	1.000000	5

V. DISCUSSION OF RESULTS

The increment that yield the minimum percent with mixed investment opportunity is 4%. Hence the optimum solution to the model is X_1= 24%, X_2 = 24%, X_3 = 24%, X_4 = 4%, and X_5 = 24%

IV. CONCLUSION

This research shows how portfolio selection of return on assets of the five selected insurance company in Nigeria was used the past financial records of each insurance company for ten years. Also, it shows how allocation of available fund by investors should be allocated to available investment open to investors. It determined that all return on asset of the insurance companies (Linkage, Niger, Mutual Benefit, LASACO and AIICO) contribute to the investor's return.

Finally, for a good product mixed or investment, equal percentage of investor's capital should be invest on Linkage insurance company, Mutual Benefit insurance company, Niger insurance company , LASACO insurance company and other remaining percent should be allocated to AIICO insurance company, so as to maximize the investor's return.

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